

Derivatives of Inverse and Logarithmic Functions

Name: _____

Date: _____

Question 1

Find the derivative of the inverse

I. $243x^5$

II. $\frac{x}{2} + \frac{3}{4}$

Question 2

Find the derivative

I. $y = \ln 3x + x$

IV. $y = \ln(\sec(\ln x))$

II. $y = \ln x^3$

III. $y = \ln(\ln x)$

Derivatives of Inverse and Logarithmic Functions

Question 3

Find the derivative

i. $y = \sqrt{x(x+1)}$

ii. $y = \frac{x+5}{x \cos x}$

Question 4

Find the derivative

i. $\ln y = e^y \sin x$

Derivatives of Inverse and Logarithmic Functions

II. $x^y = y^x$

Question 5

Find the derivative

I. $y = 2^x$

II. $y = 3^{-x}$

III. $y = \log_2 5x$

Derivatives of Inverse and Logarithmic Functions

Question 6

Find the derivative

i. $y = (x + 1)^x$

ii. $y = (\sin x)^x$

Derivatives of Inverse and Logarithmic Functions

Name: _____ **Key** _____

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Question 1

Find the derivative of the inverse

$$I. \quad 243x^5 \xrightarrow{\text{inverse}} \frac{\sqrt[5]{x}}{3} \longrightarrow \frac{1}{3} \frac{d}{dx} x^{1/5} \longrightarrow \frac{1}{3} \left(\frac{1}{5} x^{-4/5} \right) \longrightarrow \frac{1}{3} \left(\frac{1}{5x^{4/5}} \right) \longrightarrow \frac{1}{15x^{4/5}}$$

$$II. \quad \frac{x}{2} + \frac{3}{4} \xrightarrow{\text{inverse}} \frac{2x+3}{4} \longrightarrow 2x - \frac{3}{2} \longrightarrow 2$$

$$2 \frac{d}{dx} x \longrightarrow 2 \quad - \frac{d}{dx} \frac{3}{2} \longrightarrow 0$$

Question 2

Find the derivative

$$I. \quad y = \ln 3x + x$$

$$\frac{1}{3x} \cdot 3 + 1 \longrightarrow \frac{3}{3x} + 1 \longrightarrow \frac{1}{x} + 1$$

$$3 \frac{d}{dx} x \longrightarrow 3 \quad \frac{d}{dx} x \longrightarrow 1$$

$$IV. \quad y = \ln(\sec(\ln x))$$

$$\frac{1}{\sec(\ln x)} \cdot \sec(\ln x) \tan(\ln x) \cdot \frac{1}{x} \cdot 1 \longrightarrow \frac{\tan(\ln x)}{x}$$

$$\frac{d}{dx} \sec(\ln x) \longrightarrow \sec(\ln x) \tan(\ln x)$$

$$\frac{d}{dx} x \longrightarrow 1$$

$$II. \quad y = \ln x^3$$

$$\frac{1}{x^3} \cdot 3x^2 \longrightarrow \frac{3x^2}{x^3} \longrightarrow \frac{3}{x}$$

$$\frac{d}{dx} x^3 \longrightarrow 3x^2$$

$$III. \quad y = \ln(\ln x)$$

$$\frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

Derivatives of Inverse and Logarithmic Functions

Question 3

Find the derivative

i. $y = \sqrt{x(x+1)} \longrightarrow (x(x+1))^{1/2} \longrightarrow (x^2+x)^{1/2}$

$$y = u^{1/2} \quad u = x^2 + x$$

$$\frac{d}{dx} u^{1/2} \longrightarrow (1/2)u^{-1/2} \longrightarrow (1/2)(x^2+x)^{-1/2} \longrightarrow \frac{1}{2(x^2+x)^{1/2}}$$

$$\frac{d}{dx} x^2 \longrightarrow 2x \quad \frac{d}{dx} x \longrightarrow 1 \quad 2x+1$$

$$\frac{1}{2(x^2+x)^{1/2}} (2x+1) \longrightarrow \frac{2x+1}{2(x^2+x)^{1/2}}$$

ii. $y = \frac{x+5}{x \cos x}$

$$\frac{d}{dx} x \longrightarrow 1 \quad \frac{d}{dx} 5 \longrightarrow 0$$

$$\frac{d}{dx} x \longrightarrow 1 \quad \frac{d}{dx} \cos(x) \longrightarrow -\sin(x) \quad x \cdot -\sin(x) + \cos(x) \cdot 1 \longrightarrow -x \sin(x) + \cos(x)$$

$$\frac{(x \cos x)(1) - (x+5)(-x \sin(x) + \cos(x))}{(x \cos x)^2} \longrightarrow \frac{(x \cos x) - (x+5)(-x \sin(x) + \cos(x))}{(x \cos x)^2}$$

Question 4

Find the derivative

i. $\ln y = e^y \sin x \longrightarrow \frac{1}{y} \cdot 1y' = e^y \cos(x) + e^y y' \sin(x) \longrightarrow \frac{1y'}{y} = e^y \cos(x) + e^y y' \sin(x) \longrightarrow \frac{1y'}{y} - e^y y' \sin(x) = e^y \cos(x)$

$$\frac{ye^y \cos(x)}{1 - ye^y \sin(x)} \longleftarrow \frac{1 - ye^y \sin(x)}{y} = e^y \cos(x) \longleftarrow \frac{1y' - ye^y y' \sin(x)}{y} = e^y \cos(x)$$

$$\frac{d}{dx} e^y \longrightarrow e^y y' \quad \frac{d}{dx} \sin(x) \longrightarrow \cos(x) \quad e^y \cdot \cos(x) + \sin(x) \cdot e^y y' \longrightarrow e^y \cos(x) + e^y y' \sin(x)$$

$$\frac{d}{dx} y \longrightarrow 1y'$$

Derivatives of Inverse and Logarithmic Functions

$$\begin{aligned}
 \text{II. } x^y = y^x &\longrightarrow \ln x^y = \ln y^x \longrightarrow y \ln x = x \ln y \longrightarrow y' \ln x + \frac{y}{x} = \ln y + \frac{xy'}{y} \longrightarrow y' \ln x - \frac{xy'}{y} = \ln y - \frac{y}{x} \\
 &\qquad\qquad\qquad \downarrow \\
 &\qquad\qquad\qquad \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}} \longleftarrow \frac{y(x \ln(y) - y)}{x(y \ln(x) - x)}
 \end{aligned}$$

$$\frac{d}{dx} y \longrightarrow 1y' \qquad 1y' \cdot \ln x + y \cdot \frac{1}{x} \longrightarrow y' \ln x + \frac{y}{x}$$

$$\frac{d}{dx} x \longrightarrow 1 \qquad 1 \cdot \ln y + x \cdot \frac{1y'}{y} \longrightarrow \ln y + \frac{xy'}{y}$$

$$\frac{d}{dx} \ln x \longrightarrow \frac{1}{x} \cdot 1$$

$$\frac{d}{dx} \ln y \longrightarrow \frac{1}{y} \cdot 1y'$$

Question 5

Find the derivative

$$\text{I. } y = 2^x \longrightarrow 2^x \ln 2 \cdot 1 \longrightarrow 2^x \ln 2$$

$$\frac{d}{dx} x \longrightarrow 1$$

$$\text{II. } y = 3^{-x} \longrightarrow 3^{-x} \ln 3 \cdot -1 \longrightarrow -3^{-x} \ln 3$$

$$-\frac{d}{dx} x \longrightarrow -1$$

$$\text{III. } y = \log_2 5^x \longrightarrow \frac{1}{5^x \ln 2} \cdot 5 \longrightarrow \frac{5}{5^x \ln 2} \longrightarrow \frac{1}{x \ln 2}$$

$$5 \frac{d}{dx} x \longrightarrow 5$$

Derivatives of Inverse and Logarithmic Functions

Question 6

Find the derivative

$$i. \quad y = (x+1)^x \longrightarrow \ln y = \ln (x+1)^x \longrightarrow \ln y = x \ln (x+1) \longrightarrow \frac{1y'}{y} = x \cdot \frac{1}{(x+1)} + \ln (x+1)$$

↓

$$(x+1)^x \left(\frac{x}{(x+1)} + \ln (x+1) \right) \longleftarrow (y) \left(\frac{x}{(x+1)} + \ln (x+1) \right)$$

$$\frac{d}{dx} \ln y \longrightarrow \frac{1}{y} \cdot 1y' \longrightarrow \frac{1y'}{y}$$

$$\frac{d}{dx} x \longrightarrow 1 \quad \frac{d}{dx} \ln (x+1) \longrightarrow \frac{1}{(x+1)} \cdot 1 \longrightarrow \frac{1}{(x+1)}$$

$$1 \cdot \ln (x+1) + x \cdot \frac{1}{(x+1)} \longrightarrow \ln (x+1) + \frac{x}{(x+1)}$$

$$ii. \quad y = (\sin x)^x \longrightarrow \ln y = \ln (\sin x)^x \longrightarrow \ln y = x \ln (\sin x) \longrightarrow \frac{1y'}{y} = x \cdot \frac{\cos x}{\sin x} + \ln (\sin x)$$

↓

$$(\sin x)^x (x \cot x + \ln (\sin x)) \longleftarrow (y) (x \cot x + \ln (\sin x))$$

$$\frac{d}{dx} \ln y \longrightarrow \frac{1}{y} \cdot 1y' \longrightarrow \frac{1y'}{y}$$

$$\frac{d}{dx} x \longrightarrow 1 \quad \frac{d}{dx} \ln (\sin x) \longrightarrow \frac{1}{(\sin x)} \cdot \cos x \longrightarrow \frac{\cos x}{\sin x}$$

$$1 \cdot \ln (\sin x) + x \cdot \frac{\cos x}{\sin x} \longrightarrow \ln (\sin x) + x \cdot \frac{\cos x}{\sin x}$$