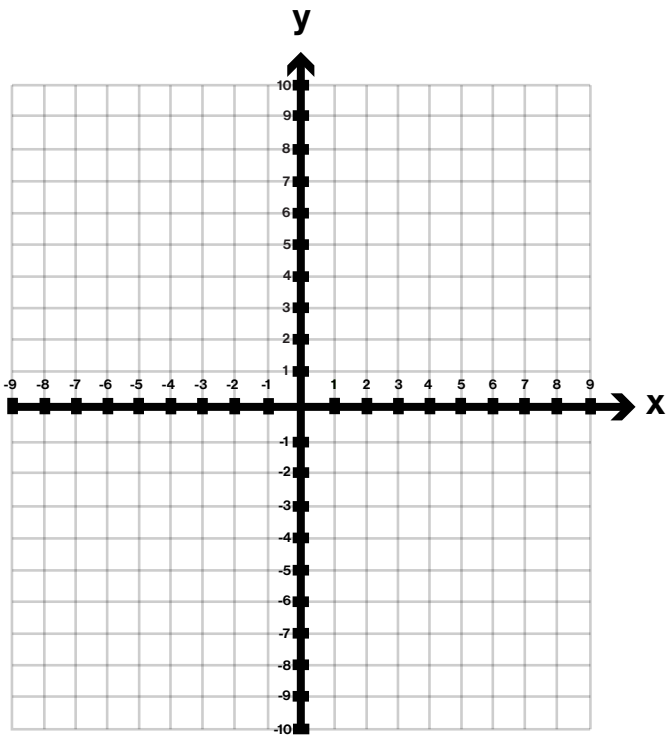
Polynomial and Rational Inequalities

Name: _____ Date: _____

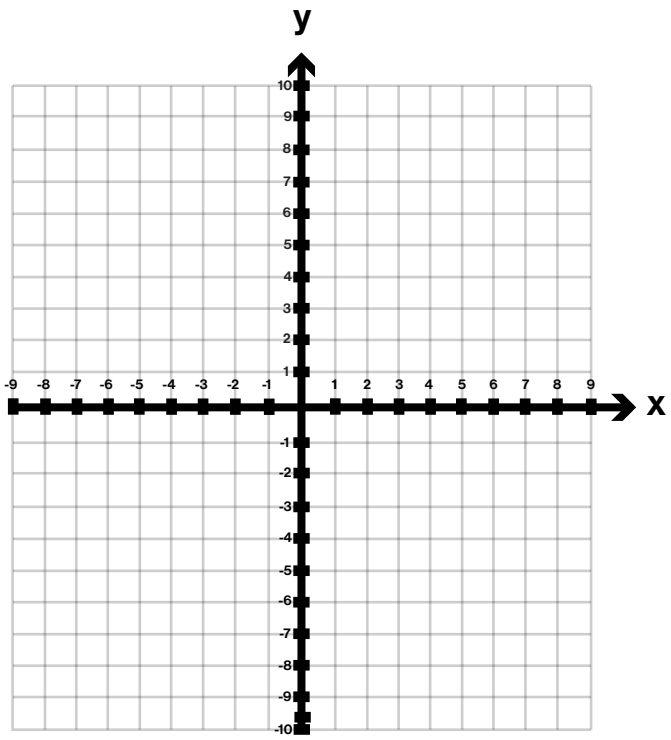
Question 1

Solve

I. $(x - 2)(x + 7) \geq 0$

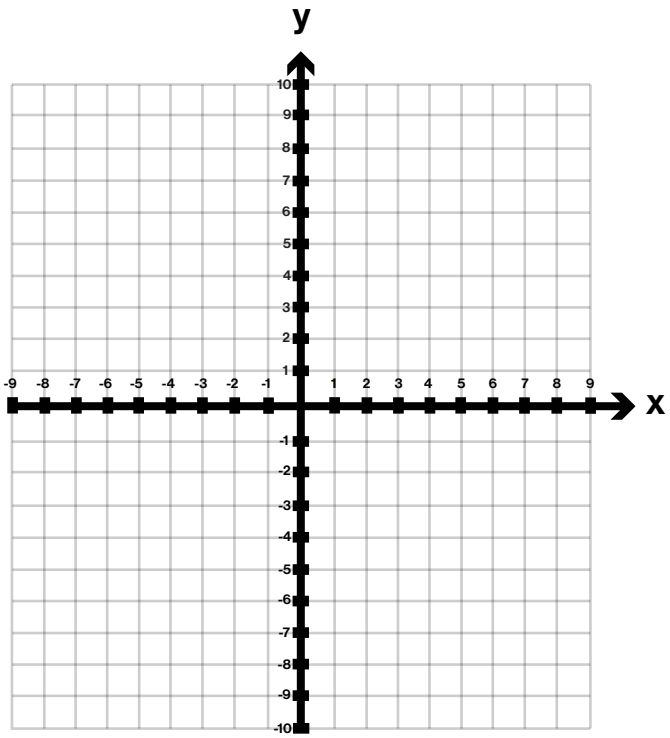


II. $(x - 2)(x + 7) \leq 0$

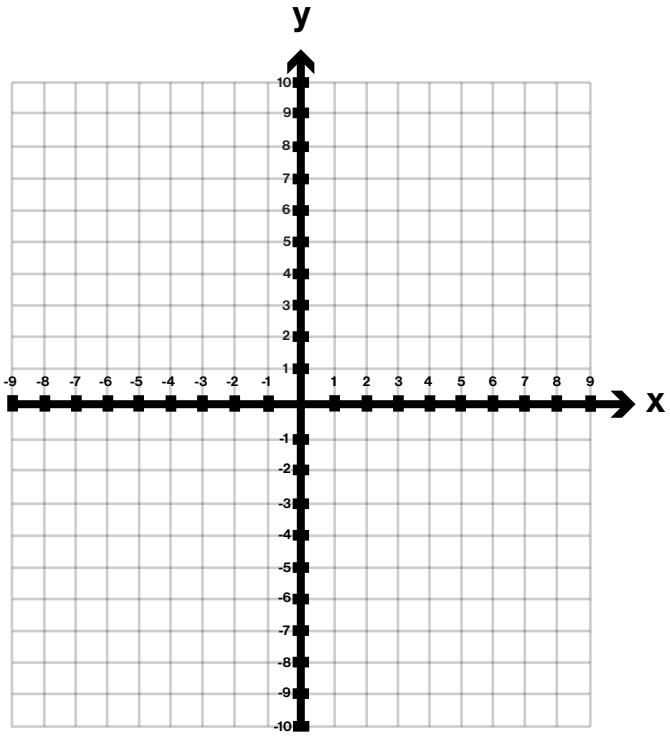


Polynomial and Rational Inequalities

III. $x^2 + 4x + 4 < 0$

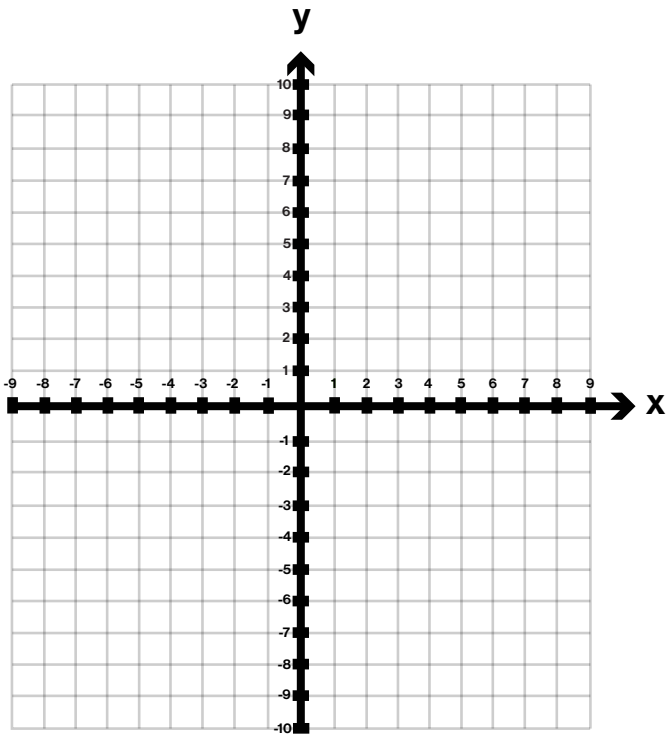


IV. $x^2 + 4x + 4 > 0$

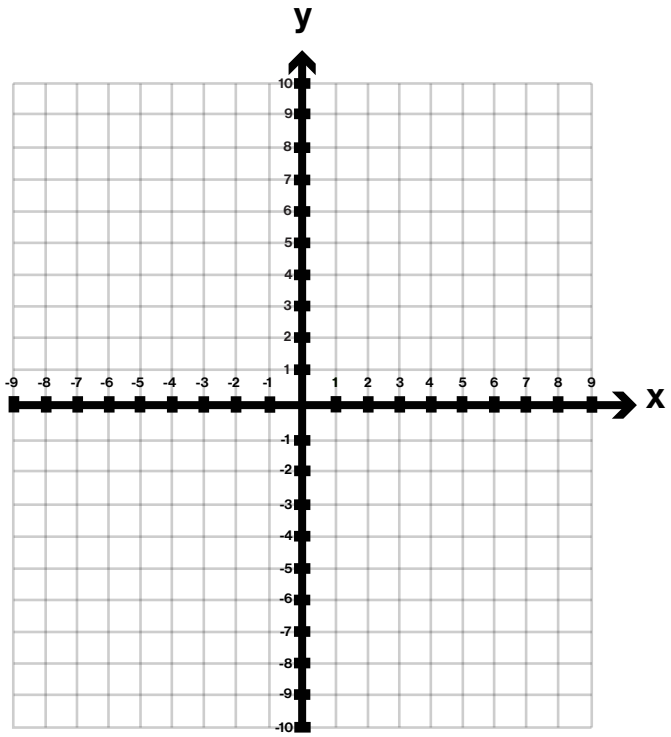


Polynomial and Rational Inequalities

V. $x^2 - 4x \leq 3$

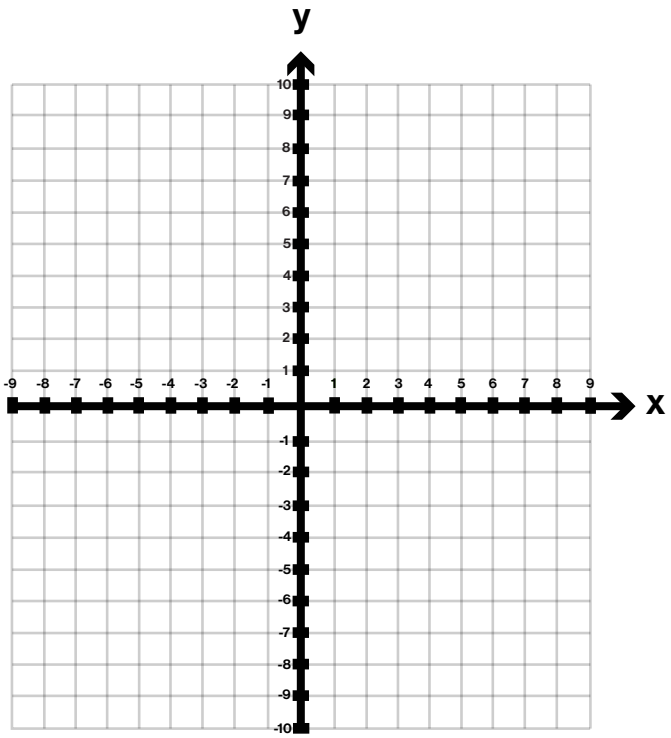


VI. $x^2 - 4x \geq 3$

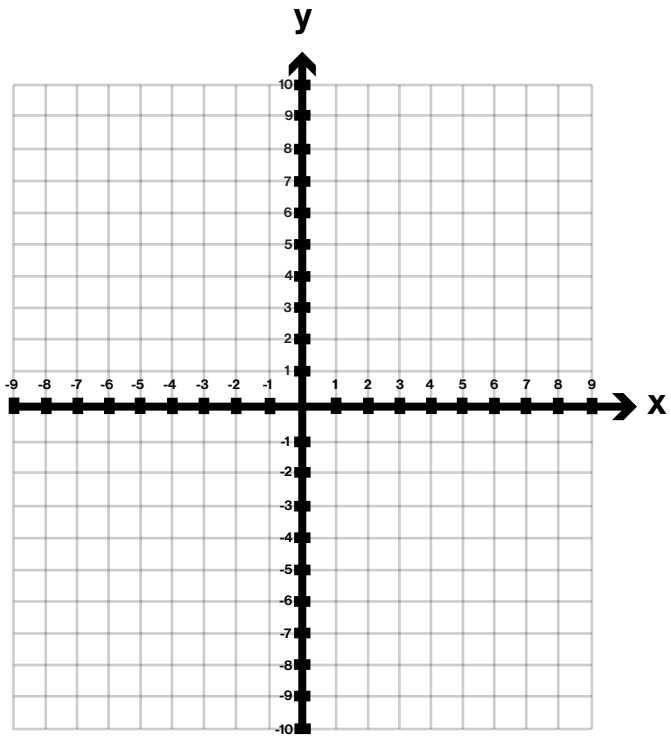


Polynomial and Rational Inequalities

VII. $5x(x+1)(x-1) > 0$

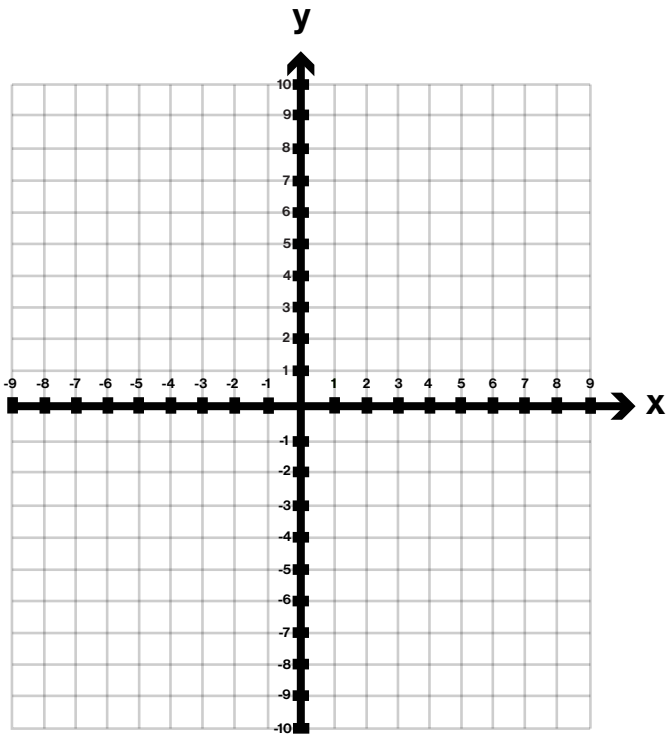


VIII. $5x(x+1)(x-1) < 0$

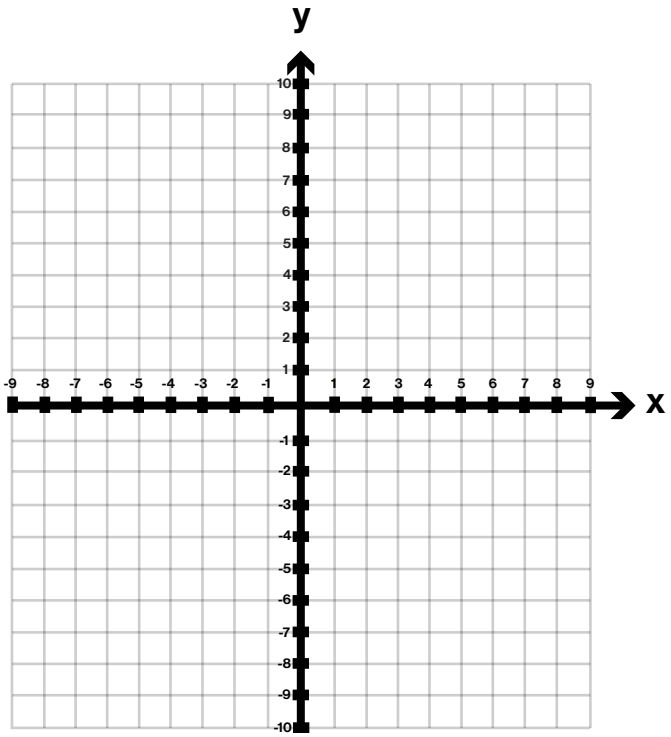


Polynomial and Rational Inequalities

IX. $(x - 1)(x + 2)(x - 4) > 0$



X. $(x - 1)(x + 2)(x - 4) < 0$



Polynomial and Rational Inequalities

XI. $\frac{1}{x-5} < 0$

XII. $\frac{x-2}{x+4} \leq 0$

XIII. $\frac{(x-2)(x+1)}{x-5} \leq 0$

Polynomial and Rational Inequalities

XIV. $\frac{x+2}{(x-2)(x+7)} \geq 0$

Polynomial and Rational Inequalities

Name: **Key** Date: _____

Question 1

Solve

I. $(x-2)(x+7) \geq 0 \longrightarrow x^2+5x-14 \geq 0$

$\downarrow$

$(x-2)(x+7)=0$

$\downarrow$

$x-2=0$

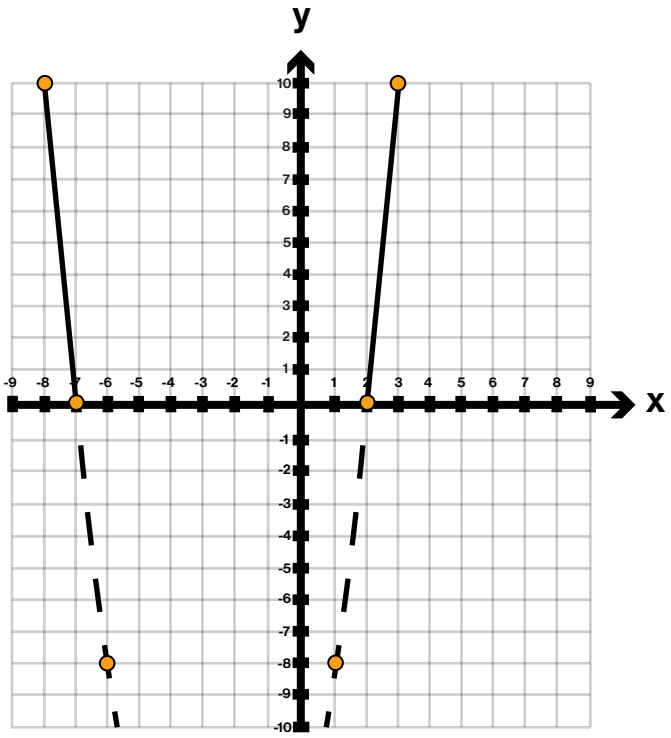
$x+7=0$

$\downarrow$

$x=2$

$x=-7$

$(-\infty, -7] \cup [2, \infty)$



II. $(x-2)(x+7) \leq 0 \longrightarrow x^2+5x-14 \leq 0$

$\downarrow$

$(x-2)(x+7)=0$

$\downarrow$

$x-2=0$

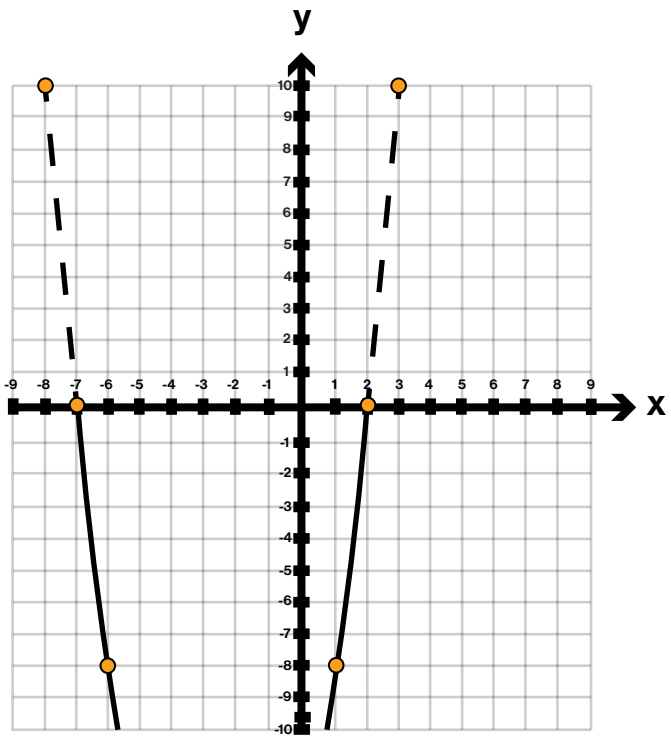
$x+7=0$

$\downarrow$

$x=2$

$x=-7$

$[-7, 2]$



Polynomial and Rational Inequalities

III. $x^2 + 4x + 4 < 0$

$\downarrow$

$x^2 + 4x + 4 = 0$

$\downarrow$

$(x + 2)(x + 2) = 0$

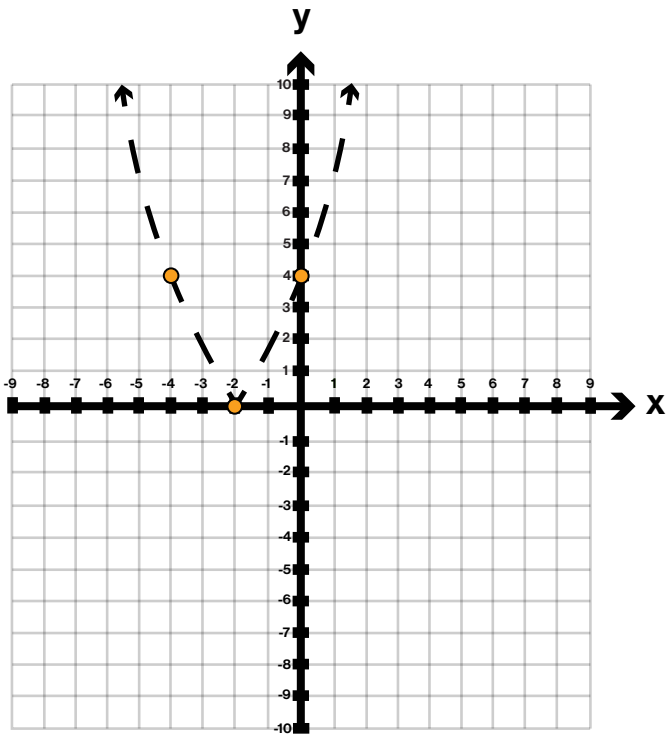
$\downarrow$

$x + 2 = 0$

$\downarrow$

$x = -2$

No Solution



IV. $x^2 + 4x + 4 > 0$

$\downarrow$

$x^2 + 4x + 4 = 0$

$\downarrow$

$(x + 2)(x + 2) = 0$

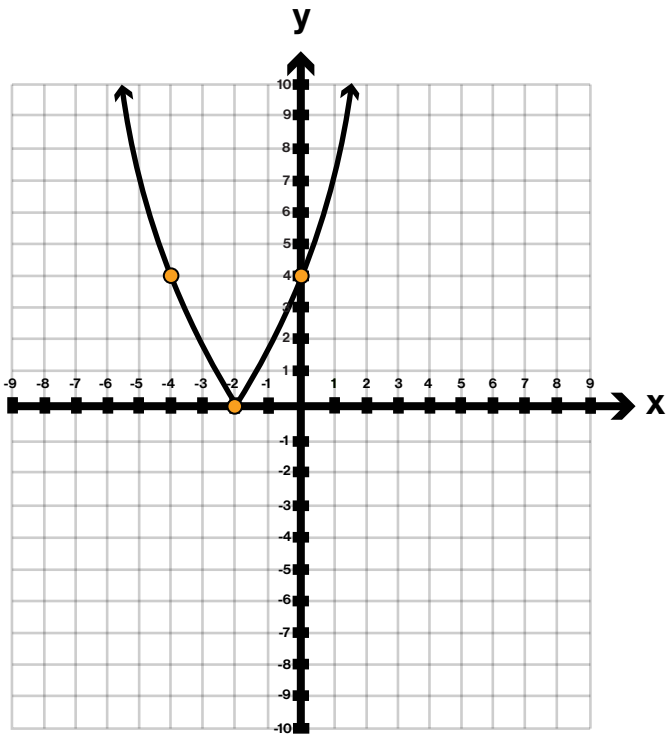
$\downarrow$

$x + 2 = 0$

$\downarrow$

$x = -2$

$(-\infty, -2) \cup (-2, \infty)$



Polynomial and Rational Inequalities

V. $x^2 - 4x \leq 3$

$\downarrow$

$x^2 - 4x = 3$

$\downarrow$

$x^2 - 4x - 3 = 0$

$\downarrow$

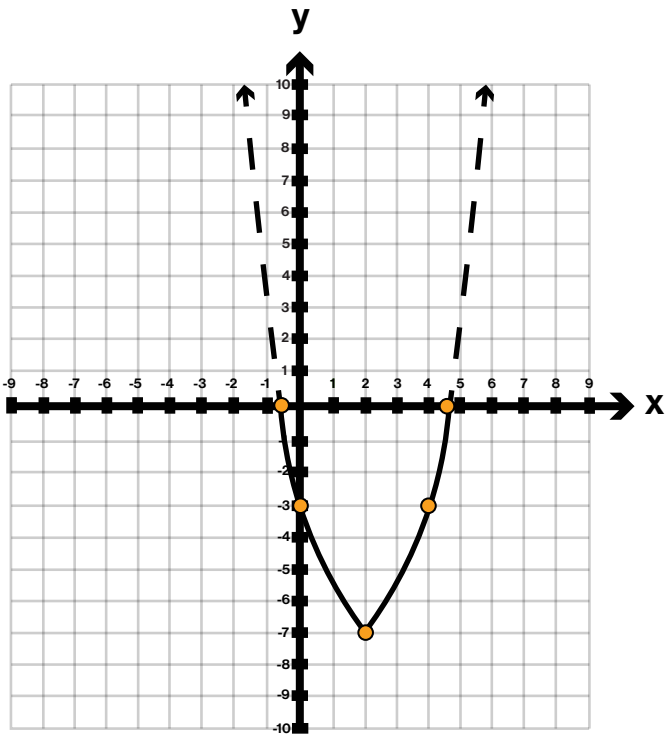
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$\downarrow$

$x = 2 + \sqrt{7}$ and $2 - \sqrt{7}$

$\downarrow$

$[2 - \sqrt{7}, 2 + \sqrt{7}]$



VI. $x^2 - 4x \geq 3$

$\downarrow$

$x^2 - 4x = 3$

$\downarrow$

$x^2 - 4x - 3 = 0$

$\downarrow$

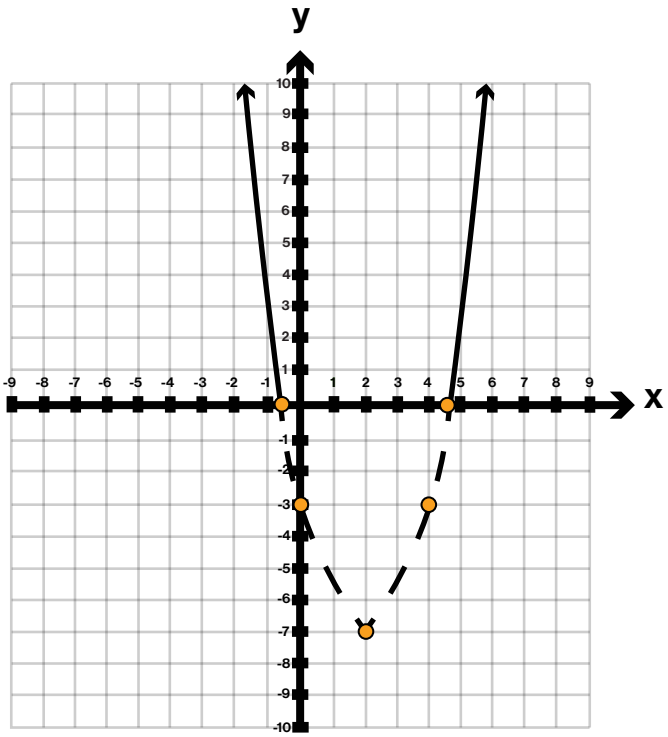
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$\downarrow$

$x = 2 + \sqrt{7}$ and $2 - \sqrt{7}$

$\downarrow$

$(-\infty, 2 - \sqrt{7}] \cup [2 + \sqrt{7}, \infty)$



Polynomial and Rational Inequalities

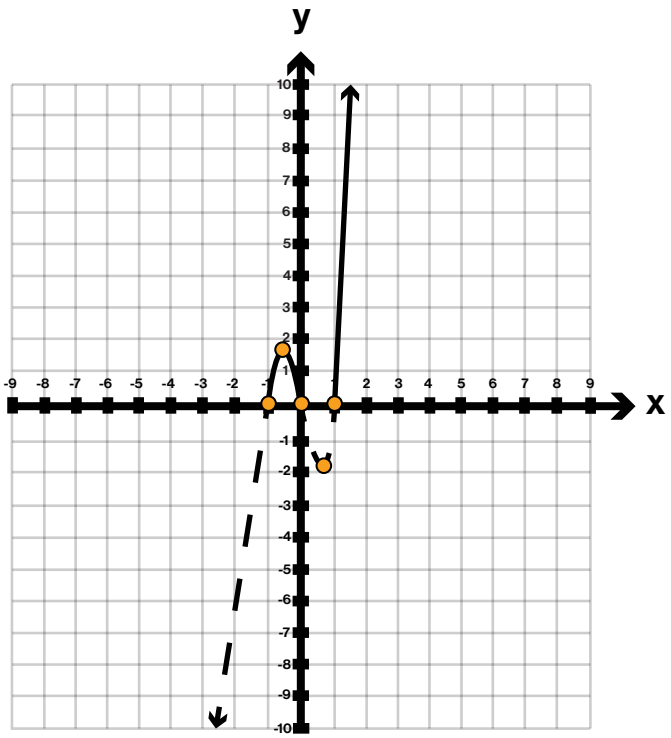
VII. $5x(x+1)(x-1) > 0$

$\downarrow$

$5x^3 - 5x > 0$

$\downarrow$

$(-1, 0) \cup (1, \infty)$



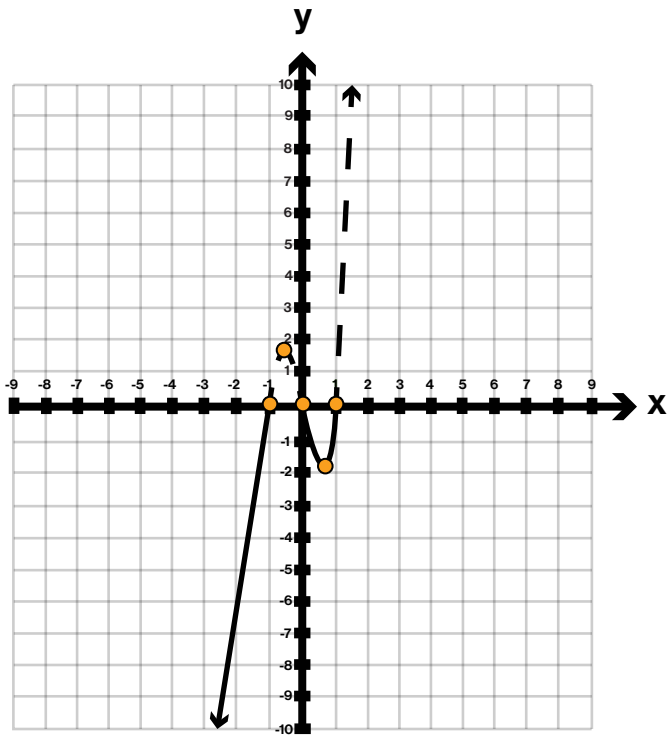
VIII. $5x(x+1)(x-1) < 0$

$\downarrow$

$5x^3 - 5x < 0$

$\downarrow$

$(-\infty, -1) \cup (0, 1)$



Polynomial and Rational Inequalities

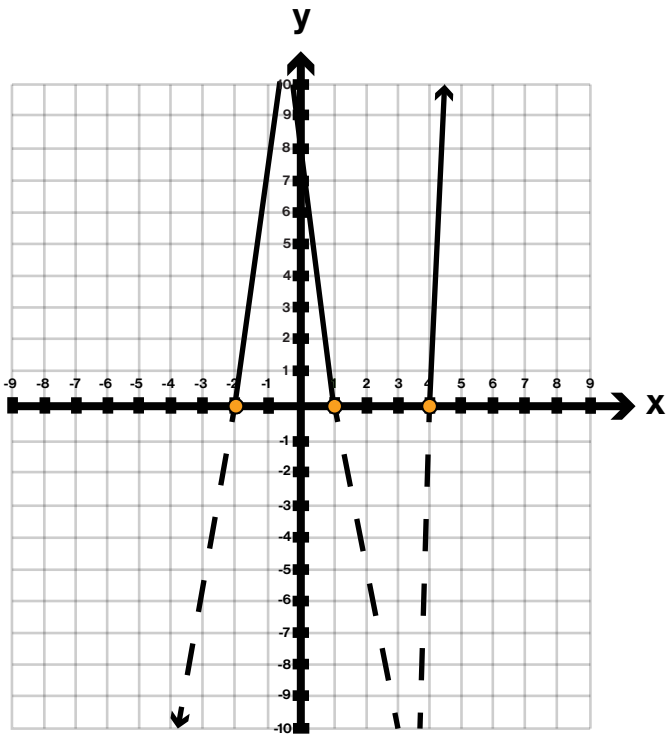
IX. $(x-1)(x+2)(x-4) > 0$

$\downarrow$

$x^3 - 3x^2 - 6x + 8 > 0$

$\downarrow$

$(-2, 1) \cup (4, \infty)$



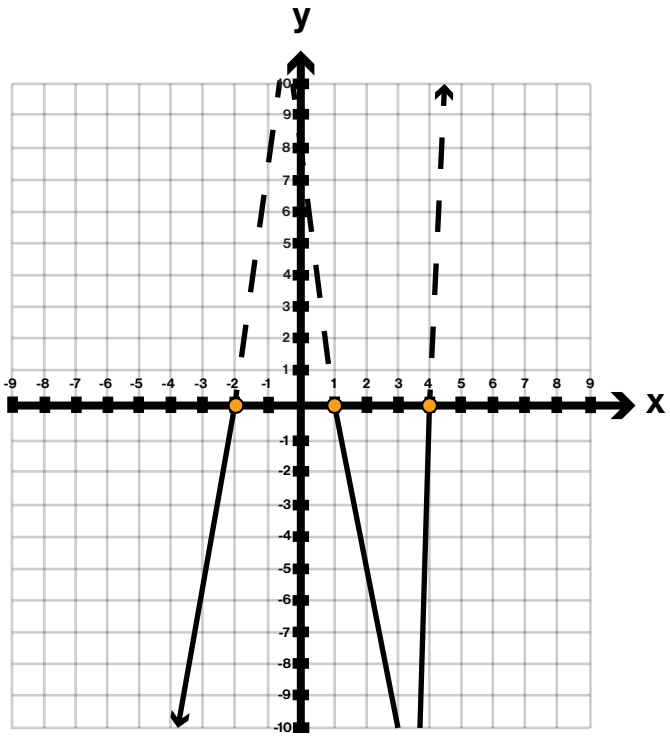
X. $(x-1)(x+2)(x-4) < 0$

$\downarrow$

$x^3 - 3x^2 - 6x + 8 < 0$

$\downarrow$

$(-\infty, -2) \cup (1, 4)$



Polynomial and Rational Inequalities

XI. $\frac{1}{x-5} < 0$

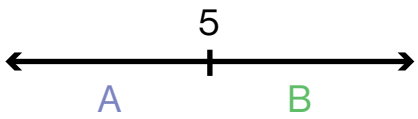
$\downarrow$

$\frac{1}{x-5} = 0 \quad x \neq 5$

$\downarrow$

$1 = 0$

No Solution



A. $\frac{1}{0-5} < 0 \longrightarrow -0.2 < 0$ True

B. $\frac{1}{7-5} < 0 \longrightarrow 0.5 < 0$ Not True

$(-\infty, 5)$

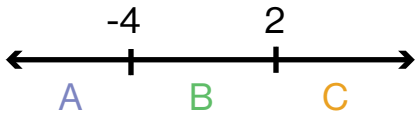
XII. $\frac{x-2}{x+4} \leq 0$

$\downarrow$

$\frac{x-2}{x+4} = 0 \quad x \neq -4$

$\downarrow$

$x = 2$



A. $\frac{-5-2}{-5+4} \leq 0 \longrightarrow 7 \leq 0$ Not True

B. $\frac{1-2}{1+4} \leq 0 \longrightarrow -0.2 \leq 0$ True

C. $\frac{7-2}{7+4} \leq 0 \longrightarrow 0.45 \leq 0$ Not True

$(-4, 2]$

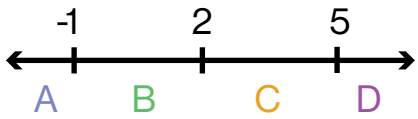
XIII. $\frac{(x-2)(x+1)}{x-5} \leq 0$

$\downarrow$

$\frac{(x-2)(x+1)}{x-5} = 0 \quad x \neq 5$

$\downarrow$

$x = 2$ and -1



A. $\frac{(-3-2)(-3+1)}{-3-5} \leq 0 \longrightarrow -1.25 \leq 0$ True

B. $\frac{(1-2)(1+1)}{1-5} \leq 0 \longrightarrow 0.5 \leq 0$ Not True

C. $\frac{(3-2)(3+1)}{3-5} \leq 0 \longrightarrow -2 \leq 0$ True

D. $\frac{(7-2)(7+1)}{7-5} \leq 0 \longrightarrow 20 \leq 0$ Not True

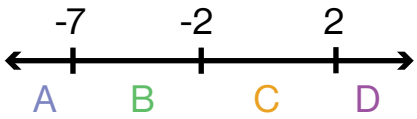
$(-\infty, -1] \cup [2, 5)$

Polynomial and Rational Inequalities

XIV. $\frac{x+2}{(x-2)(x+7)} \geq 0$

$\downarrow$

$\frac{x+2}{(x-2)(x+7)} = 0 \quad x \neq 2 \text{ and } -7$



$\downarrow$

$x = -2$

A.

$\frac{-8+2}{(-8-2)(-8+7)} \geq 0 \longrightarrow -0.6 \geq 0$ Not True

B.

$\frac{-3+2}{(-3-2)(-3+7)} \geq 0 \longrightarrow 0.05 \geq 0$ True

C.

$\frac{1+2}{(1-2)(1+7)} \geq 0 \longrightarrow -0.375 \geq 0$ Not True

D.

$\frac{5+2}{(5-2)(5+7)} \geq 0 \longrightarrow 0.194 \geq 0$ True

$(-7, -2] \cup (2, \infty)$