

# Derivatives of Functions

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Question 1

Find the derivative and the derivative values

1.  $f(x) = (x - 1)^2 + 1$

$f'(-1) =$

$f'(0) =$

$f'(2) =$

# Derivatives of Functions

## Question 2

Find the derivative

i.  $\frac{dy}{dx}$      $y = \frac{1}{-\sqrt{x^2 - 1}}$

# Derivatives of Functions

# Derivatives of Functions

## Question 3

Find the derivative and the slope of the tangent line

i.  $f(x) = x + \frac{9}{x}$      $x = -3$

# Derivatives of Functions

## Question 4

Find the derivative

i.  $\frac{dy}{dx} \Big|_{x=-1}$       $y = 1 - 3x^2$

# Derivatives of Functions

## Question 5

Find the derivative and the equation of the tangent line

i.  $y = 1 + \sqrt{4 - x}$       (3, 2)

# Derivatives of Functions

# Derivatives of Functions

## Question 6

Find the derivative (alternative formula)

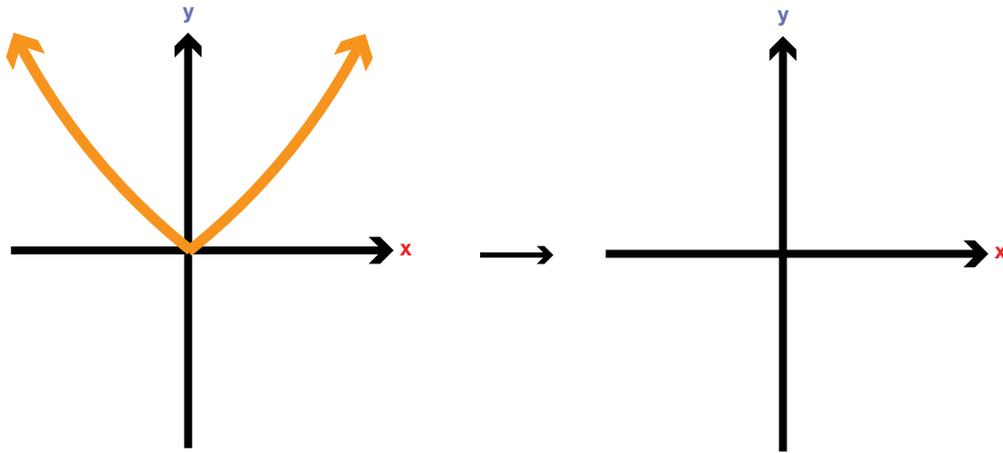
i.  $f(x) = \frac{1}{x+2}$

# Derivatives of Functions

## Question 7

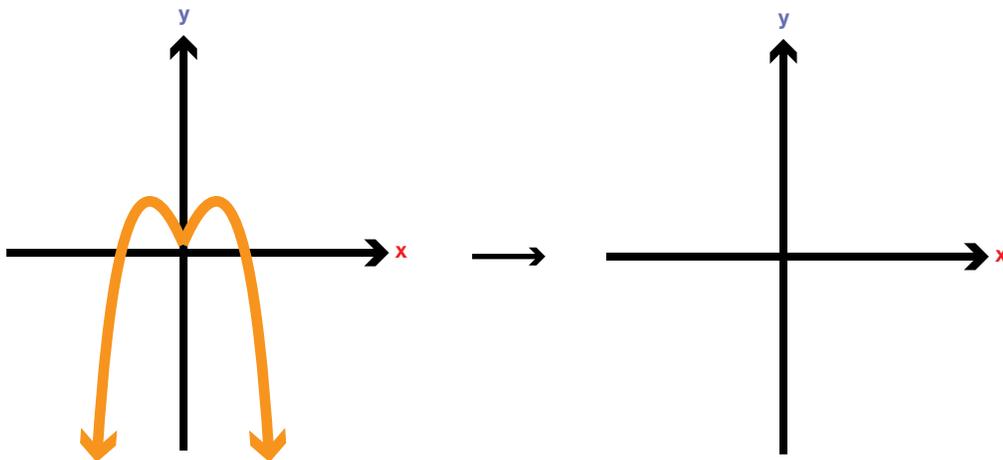
Sketch the Derivative

I.



$$f(x) = \frac{3}{4}x^{4/3}$$

II.



$$f(x) = -x^4 + x^2$$

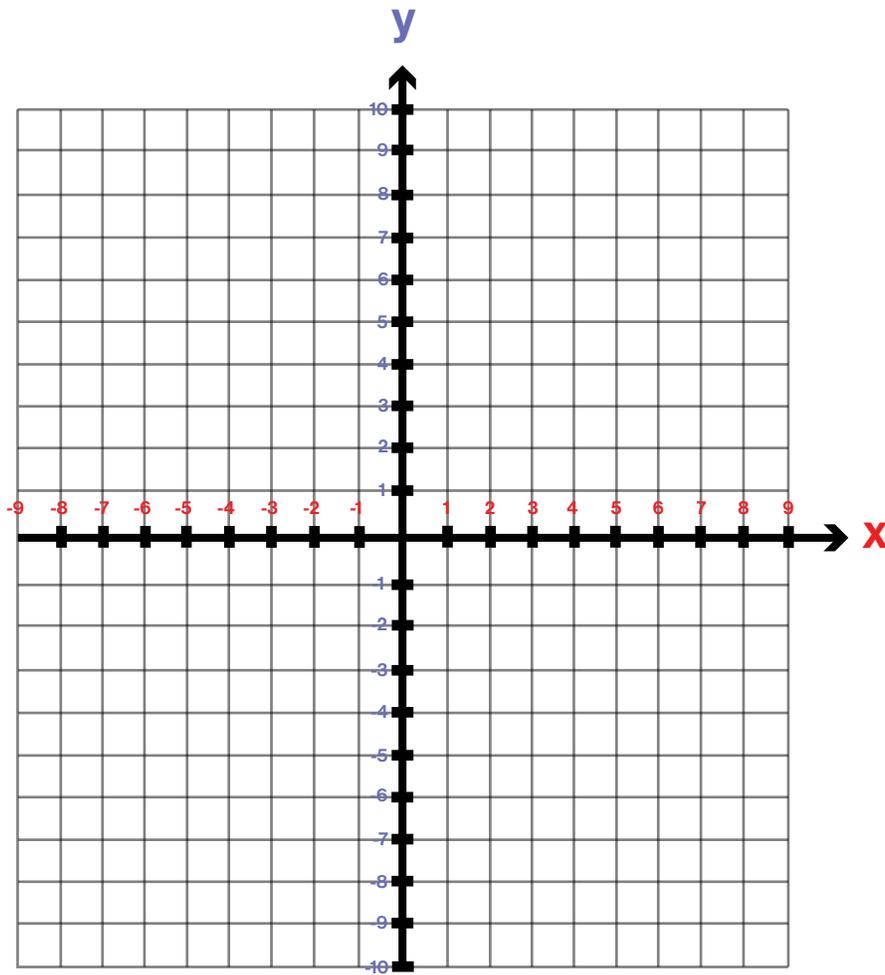
# Derivatives of Functions

## Question 8

Determine if the piecewise function is differentiable at  $x = 0$

i.

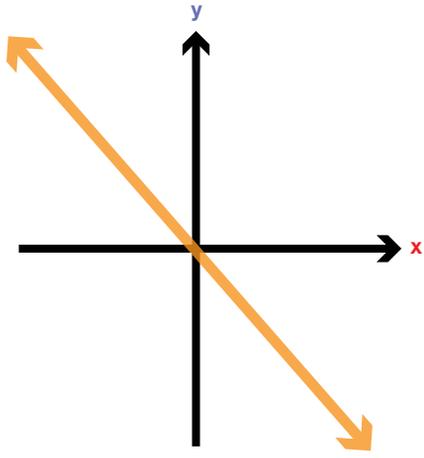
$$f(x) = \begin{cases} 2x - 1 & x \geq 0 \\ x^2 + 2x + 7 & x < 0 \end{cases}$$



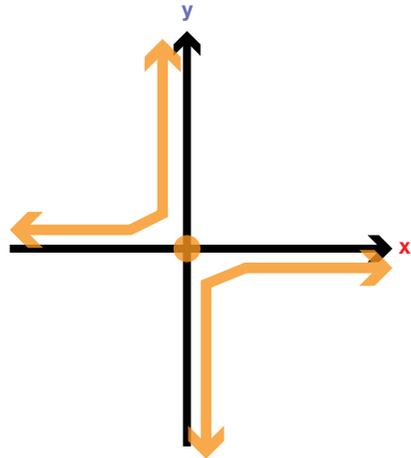
# Derivatives of Functions

Question 9  
Define the graph

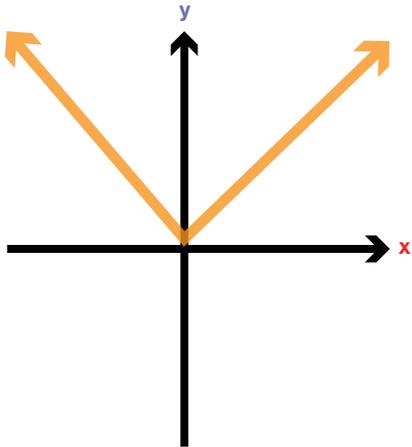
I.



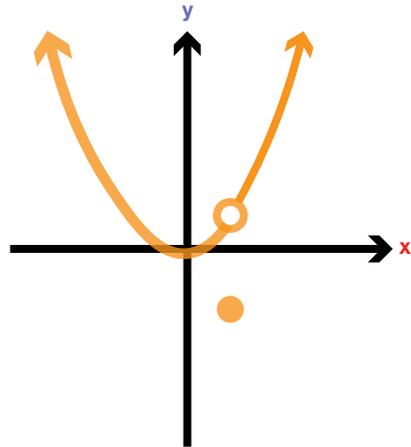
II.



III.



IV.



# Derivatives of Functions

Name: \_\_\_\_\_ **Key** \_\_\_\_\_

Date: \_\_\_\_\_

## Question 1

Find the derivative and the derivative values

i.  $f(x) = (x - 1)^2 + 1$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{((x+h-1)^2 + 1) - ((x-1)^2 + 1)}{h} &\longrightarrow \frac{2hx + h^2 - 2h}{h} \longrightarrow \frac{\cancel{h}(2x+h-2)}{\cancel{h}} \\ &\qquad\qquad\qquad \downarrow \\ 2(x-1) \longleftarrow 2x-2 \longleftarrow 2x+(0)-2 \longleftarrow 2x+h-2 \end{aligned}$$

$$f'(-1) = 2((-1) - 1) = -4$$

$$f'(0) = 2((0) - 1) = -2$$

$$f'(2) = 2((2) - 1) = 2$$

# Derivatives of Functions

## Question 2

Find the derivative

$$1. \frac{dy}{dx} \quad y = \frac{1}{\sqrt{x^2-1}}$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{(x+h)^2-1}} - \frac{1}{\sqrt{x^2-1}}}{h} &\longrightarrow \frac{\frac{\sqrt{x^2-1} - \sqrt{(x+h)^2-1}}{(\sqrt{(x+h)^2-1})(\sqrt{x^2-1})}}{h} \\ &\downarrow \\ \frac{\sqrt{x^2-1} - \sqrt{(x+h)^2-1}}{h(\sqrt{(x+h)^2-1})(\sqrt{x^2-1})} & \cdot \frac{\sqrt{x^2-1} + \sqrt{(x+h)^2-1}}{\sqrt{x^2-1} + \sqrt{(x+h)^2-1}} \\ &\downarrow \\ \frac{-2hx - h^2}{h(\sqrt{(x+h)^2-1})(\sqrt{x^2-1})(\sqrt{x^2-1} + \sqrt{(x+h)^2-1})} & \\ &\downarrow \\ \frac{-2x - h}{(\sqrt{(x+h)^2-1})(\sqrt{x^2-1})(\sqrt{x^2-1} + \sqrt{(x+h)^2-1})} & \end{aligned}$$

# Derivatives of Functions

$$\frac{-2x - (0)}{(-\sqrt{(x+(0))^2-1})(-\sqrt{x^2-1})(-\sqrt{x^2-1} + \sqrt{(x+(0))^2-1})}$$

$$\downarrow$$

$$\frac{-2x}{(-\sqrt{(x+1)(x-1)})(-\sqrt{(x+1)(x-1)})(-\sqrt{x^2-1} + \sqrt{(x+(0))^2-1})}$$

$$\downarrow$$

$$\frac{-2x}{((x+1)(x-1))(-\sqrt{x^2-1} + \sqrt{(x+(0))^2-1})}$$

$$\downarrow$$

$$\frac{-2x}{(x^2-1)(-\sqrt{x^2-1} + \sqrt{(x+(0))^2-1})}$$

$$\downarrow$$

$$\frac{-2x}{(x^2-1)((x^2-1)^{1/2} + (x^2-1)^{1/2})}$$

$$\downarrow$$

$$\frac{-2x}{(x^2-1)(2(x^2-1)^{1/2})}$$

$$\downarrow$$

$$\frac{\cancel{-2}x}{2(x^2-1)^{3/2}}$$

$$\downarrow$$

$$\frac{-x}{(x^2-1)^{3/2}}$$

# Derivatives of Functions

## Question 3

Find the derivative and the slope of the tangent line

i.  $f(x) = x + \frac{9}{x}$      $x = -3$

$$\lim_{h \rightarrow 0} \frac{\left( (x+h) + \frac{9}{(x+h)} \right) - \left( x + \frac{9}{x} \right)}{h}$$



$$\frac{\frac{h^2 + x^2 + 2hx + 9}{x+h} - \frac{x^2 + 9}{x}}{h}$$



$$\frac{\frac{h(hx + x^2 - 9)}{x(x+h)}}{h}$$



$$\frac{\cancel{h}(hx + x^2 - 9)}{x(x+h)} \cdot \frac{1}{\cancel{h}} \longrightarrow \frac{hx + x^2 - 9}{x(x+h)} \longrightarrow \frac{(0)x + x^2 - 9}{x(x+(0))} \longrightarrow \frac{x^2 - 9}{x^2}$$



$$1 - \frac{9}{x^2}$$

$$f'(-3) = 1 - \frac{9}{(-3)^2} = 0$$

# Derivatives of Functions

## Question 4

Find the derivative

i.  $\left. \frac{dy}{dx} \right|_{x=-1}$       $y = 1 - 3x^2$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(1 - 3(x+h)^2) - (1 - 3x^2)}{h} &\longrightarrow \frac{-6hx - 3h^2}{h} \longrightarrow \frac{\cancel{h}(-6x - 3h)}{\cancel{h}} \longrightarrow -6x - 3(0) \\ &\qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad -6x \\ &\qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \downarrow \\ \left. \frac{dy}{dx} \right|_{x=-1} &= 6 \qquad \longleftarrow -6(-1) \end{aligned}$$

# Derivatives of Functions

## Question 5

Find the derivative and the equation of the tangent line

$$y = 1 + \sqrt{4-x} \quad (3, 2)$$

$$\frac{(1 + \sqrt{4-(x+h)}) - (1 + \sqrt{4-x})}{h} \rightarrow \frac{\sqrt{4-(x+h)} - \sqrt{4-x}}{h}$$

$$\downarrow$$
$$\frac{\sqrt{4-x-h} - \sqrt{4-x}}{h}$$

$$\downarrow$$
$$\frac{\sqrt{4-x-h} - \sqrt{4-x}}{h} \cdot \frac{\sqrt{4-x-h} + \sqrt{4-x}}{\sqrt{4-x-h} + \sqrt{4-x}}$$

$$\downarrow$$
$$\frac{(4-x-h) - (4-x)}{h (\sqrt{4-x-h} + \sqrt{4-x})}$$

$$\downarrow$$
$$\frac{-h}{h (\sqrt{4-x-h} + \sqrt{4-x})}$$

# Derivatives of Functions

$$\frac{-1}{-\sqrt{4-x-h} + \sqrt{4-x}}$$

↓

$$\frac{-1}{-\sqrt{4-x-(0)} + \sqrt{4-x}}$$

↓

$$\frac{-1}{2 - \sqrt{4-x}}$$

$$f'(3) = \frac{-1}{2 - \sqrt{4-(3)}} = \frac{-1}{2}$$

at point (3, 2)

$$y - (2) = (-1/2)(x - (3)) \longrightarrow y = \frac{-1}{2}x + \frac{7}{2} \text{ Tangent Line}$$

# Derivatives of Functions

## Question 6

Find the derivative (alternative formula)

i.  $f(x) = \frac{1}{x+2}$

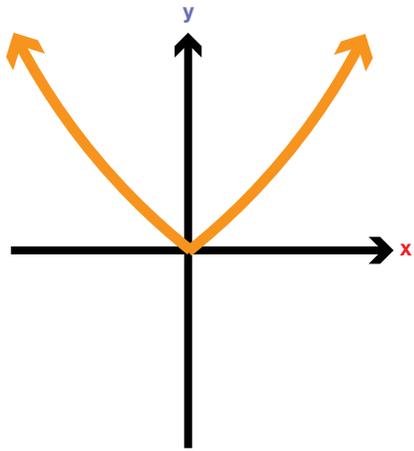
$$\begin{aligned}
 \lim_{z \rightarrow x} f'(x) &= \frac{\frac{1}{z+2} - \frac{1}{x+2}}{z-x} \longrightarrow \frac{\frac{x-z}{(x+2)(z+2)}}{z-x} \longrightarrow \frac{x-z}{(x+2)(z+2)} \cdot \frac{1}{z-x} \\
 &\qquad\qquad\qquad \downarrow \\
 &= \frac{-z+x}{(z-x)(x+2)(z+2)} \longleftarrow \frac{x-z}{(z-x)(x+2)(z+2)} \longleftarrow \frac{x-z}{(z-x)(x+2)(z+2)} \\
 &\qquad\qquad\qquad \downarrow \\
 &= \frac{-\cancel{(z-x)}}{\cancel{(z-x)}(x+2)(z+2)} \longrightarrow \frac{-1}{(x+2)(z+2)} \longrightarrow \frac{-1}{(x+2)((x)+2)} \longrightarrow \frac{-1}{(x+2)^2}
 \end{aligned}$$

# Derivatives of Functions

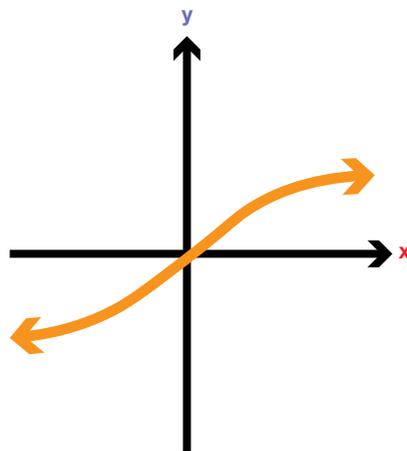
## Question 7

Sketch the Derivative

I.

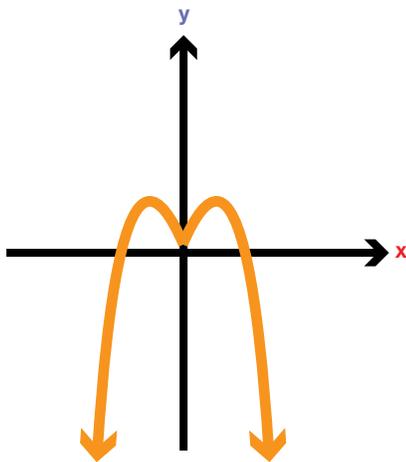


$$f(x) = \frac{3}{4}x^{4/3}$$

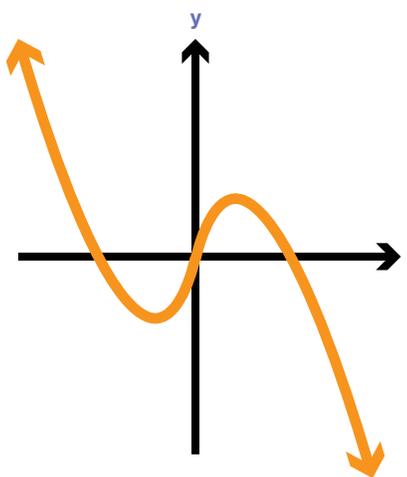


$$f'(x) = x^{1/3}$$

II.



$$f(x) = -x^4 + x^2$$



$$f'(x) = -4x^3 + 2x$$

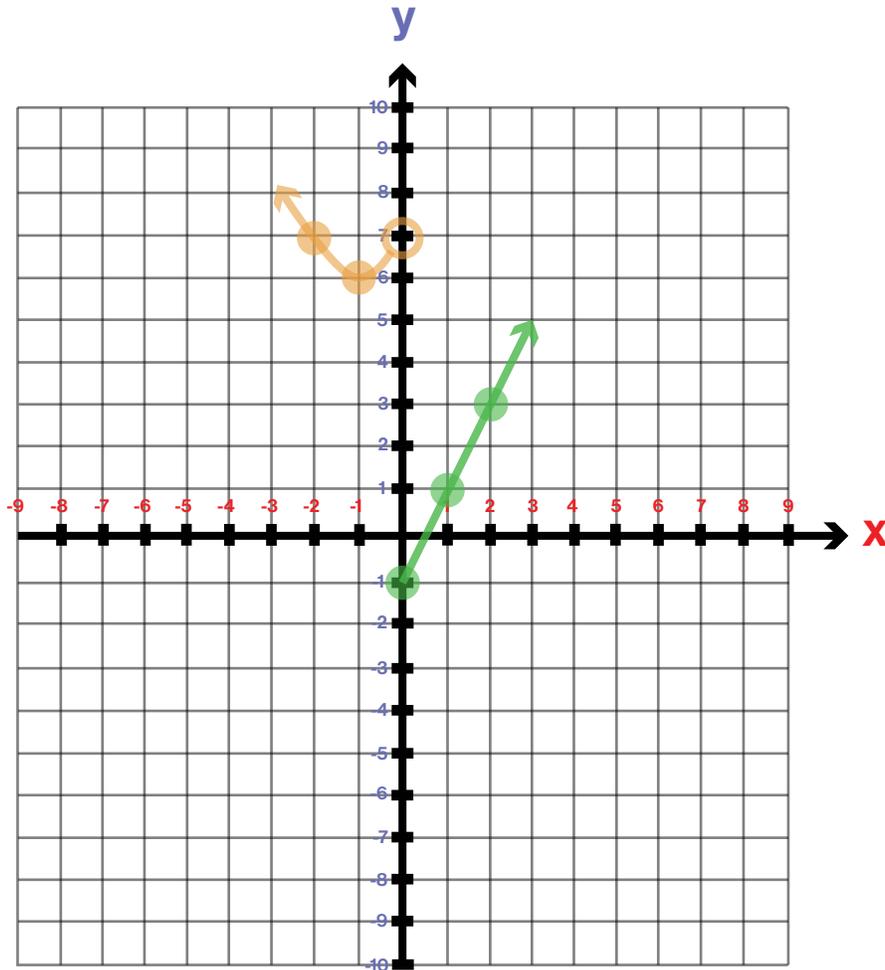
# Derivatives of Functions

## Question 8

Determine if the piecewise function is differentiable at  $x = 0$

i.

$$f(x) = \begin{cases} 2x - 1 & x \geq 0 \\ x^2 + 2x + 7 & x < 0 \end{cases}$$



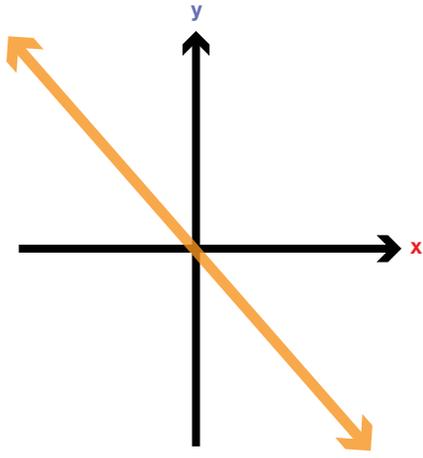
NOT differentiable at  $x = 0$

# Derivatives of Functions

## Question 9

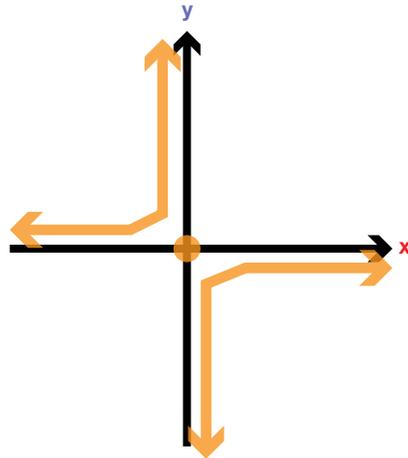
Define the graph

I.



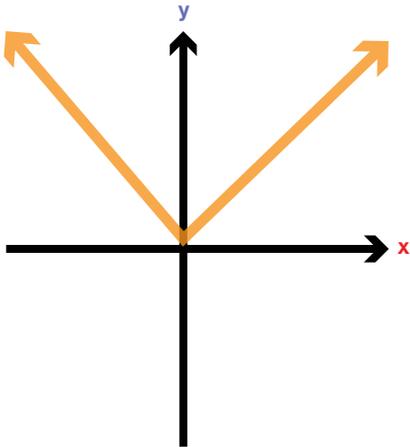
Differentiable

II.



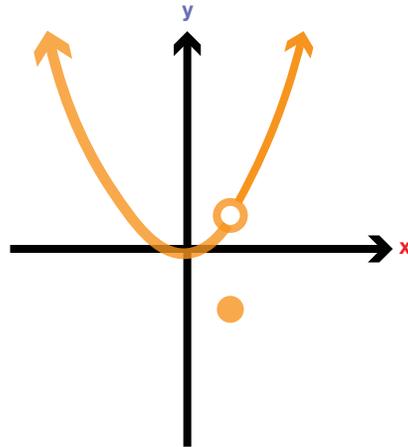
Not Continuous or Differentiable

III.



Continuous but not Differentiable

IV.



Not Continuous or Differentiable