Radical Rules

index → $\sqrt[x]{a}$ ← radical symbol
← radicand

$$\sqrt[x]{a} = b \quad \text{where} \quad b^x = a$$

Example:

$$\sqrt[2]{81} = 9 \quad \text{where} \quad 9^2 = 81$$

Exponents within Radicals

An exponent attached to the radicand can be transferred outside (Power of a Radicand Rule).

$$\sqrt[x]{a^y} = \left(\sqrt[x]{a}\right)^y$$

Example:

$$\sqrt[2]{9^3} = \left(\sqrt[2]{9}\right)^3 = 3^3 = 27$$

Radical Rules

Radicals within Radicals

Radicals within radicals can have their indices multiplied (Power Rule).

$$\sqrt[y]{\sqrt[x]{a}} = \sqrt[x \cdot y]{a}$$

Example:

$$\sqrt[3]{\sqrt[2]{64}} = \sqrt[2 \cdot 3]{64} = \sqrt[6]{64} = 2$$

Equivalent Index and Exponent

If the exponent connected to the radicand is equivalent to the index, the root is equivalent to the radicand.

$$\sqrt[x]{a^{x}} = a$$

Example:

$$\sqrt[3]{7^{3}} = 7$$

Radical Rules

Negative Radicands Within Parentheses with Equivalent Index and Exponent

(1) If the index is odd, the root will be negative.
(2) If the index is even, the root will be positive.

$$\sqrt[x]{(-a)^{x}} \quad \text{OR} \quad \sqrt[-x]{(-a)^{-x}}$$

Example (1):

$$\sqrt[3]{(-64)^{3}} = -64$$

Example (2):

$$\sqrt[2]{(-64)^{2}} = 64$$

Radical Rules

Multiplying Radicals with the Same Indices

If the indices are the same, the radicands are multiplied under the same radical symbol (Product Rule).

$$\sqrt[x]{a} \cdot \sqrt[x]{b} = \sqrt[x]{a \cdot b}$$

Example:

$$\sqrt[2]{81} \cdot \sqrt[2]{64} = \sqrt[2]{81 \cdot 64}$$

$$= \sqrt[2]{5{,}184} = 72$$

Dividing Radicals with the Same Indices

Fractions under the same radical symbol can be split apart so that the numerator and denominator are both under their own individual radical symbol (Quotient Rule).

$$\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$$

Example:

$$\sqrt[2]{\frac{4}{25}} = \frac{\sqrt[2]{4}}{\sqrt[2]{25}} = \frac{2}{5} = 0.4$$

Radical Rules

Index of One

An index of one will result in the root equaling the radicand.

$$\sqrt[1]{a} = a$$

Example:

$$\sqrt[1]{3} = 3$$

Radical Rules

Using a Calculator

Beyond square roots memorized from multiplication facts, a calculator is required to evaluate most radical equations.

Example:

$$\sqrt[2]{88} = 9.380...$$

Example:

$$\sqrt[3]{33} = 3.207...$$

Example:

$$\sqrt[-2]{2} = 0.707...$$

Radical Rules

$a \neq 0 \quad b \neq 0$

Product Rule

$$\sqrt[x]{a} \cdot \sqrt[x]{b} = \sqrt[x]{a \cdot b}$$

Quotient Rule

$$\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$$

Power Rule

$$\sqrt[y]{\sqrt[x]{a}} = \sqrt[x \cdot y]{a}$$

Power of a Radicand Rule

$$\sqrt[x]{a^{y}} = \left(\sqrt[x]{a}\right)^{y}$$