

Radicals Practice

Name: _____

Date: _____

Question 1

Simplify

I. $-\sqrt{16x^2}$

II. $-\sqrt{(-7x)^2}$

III. $-\sqrt{(8-x)^2}$

IV. $-\sqrt{x^2 + 16x + 64}$

V. $-\sqrt{x^{22}}$

Question 2

Simplify

I. $-\sqrt{144}$

II. $-\sqrt{\frac{36}{49}}$

III. $-\sqrt{100}$

IV. $-\sqrt{0.0016}$

Question 3

Find the function value

I. $f(x) = -\sqrt{5x + 10}$

$$f(2) = -\sqrt{5(2) + 10}$$

$$f(3) = -\sqrt{5(3) + 10}$$

$$f(7) = -\sqrt{5(7) + 10}$$

Radicals Practice

Question 4

Simplify

I. $\sqrt[3]{-1}$

II. $-\sqrt[3]{64}$

III. $\sqrt[3]{-64x^3}$

Question 5

Simplify

I. $-\sqrt[4]{256}$

II. $\sqrt[6]{x^6}$

III. $\sqrt[5]{\frac{-1}{32}}$

IV. $\sqrt[4]{(6x)^4}$

V. $\sqrt[12]{(-10)^{12}}$

Question 6

Simplify

I. $-\sqrt{(x-2)^8}$

II. $-\sqrt{(x+2)^{10}}$

Question 7

Determine the domain

I. $f(x) = \sqrt[5]{2x+7}$

II. $f(x) = \sqrt{x+8}$

III. $f(x) = \sqrt[3]{2x-6}$

IV. $f(x) = -\sqrt[6]{5x+2}$

V. $f(x) = 9 + \sqrt[6]{x^6}$

Radicals Practice

Question 8

Convert to radical form

I. $x^{1/3}$

II. $36^{1/5}$

III. $x^{5/6}$

IV. $(9x^6)^{3/2}$

Question 9

Convert to exponent form

I. $\sqrt[3]{18}$

II. $\sqrt[5]{x^4}$

III. $-\sqrt{24}$

IV. $\sqrt[7]{x^3y^2z^3}$

V. $(\sqrt[6]{2x^5y})^7$

VI. $\frac{3x}{\sqrt[5]{y^2}}$

Question 10

Convert to radical form

I. $6^{-1/3}$

II. $(\frac{1}{15})^{-3/4}$

III. $\frac{8x}{y^{-3/5}}$

IV. $(\frac{2xy}{3z})^{-5/6}$

Radicals Practice

Question 11

Simplify

I. $\sqrt[9]{y^3}$

II. $(\sqrt[7]{xy})^{14}$

III. $\sqrt{\sqrt[5]{x}}$

IV. $\sqrt[3]{\sqrt[4]{xy}}$

V. $\sqrt[4]{(xy)^{12}}$

Question 12

Multiply

I. $\sqrt{10} \sqrt{3}$

II. $\sqrt[3]{2} \sqrt[3]{3}$

III. $\sqrt{5x} \sqrt{6y}$

IV. $\sqrt[5]{9x^2} \sqrt[5]{2x}$

V. $\sqrt[3]{9x^2} \sqrt[3]{2y}$

VI. $\sqrt[4]{y-1} \sqrt[4]{y^2+y+1}$

VII. $\sqrt{\frac{7x}{6}} \sqrt{\frac{5}{y}}$

Radicals Practice

Question 13

Simplify

I. $-\sqrt{12}$

II. $-\sqrt{300}$

III. $-\sqrt{36x^4y}$

IV. $\sqrt[3]{-32x^6}$

V. $\sqrt[5]{-32x^7y^{11}}$

VI. $-\sqrt{x^5y^6z^{10}}$

Question 14

Multiply

I. $-\sqrt{10} \cdot -\sqrt{5}$

II. $-\sqrt{24x^5} \cdot -\sqrt{24x^5}$

III. $3\sqrt{5x^7} \cdot 2\sqrt{15x^3}$

IV. $\sqrt[5]{x^3(y-z)^4} \cdot \sqrt[5]{x^7(y-z)^4}$

Radicals Practice

Question 15

Simplify

I. $\sqrt{\frac{49}{100}}$

II. $\sqrt{\frac{25x^5}{y^6}}$

III. $\sqrt[3]{\frac{27x^4}{8y^3}}$

Question 16

Divide

I. $\frac{\sqrt{18x}}{\sqrt{2x}}$

II. $\frac{\sqrt[3]{189y^5z^7}}{\sqrt[3]{7y^2z^2}}$

III. $\frac{\sqrt[3]{y^3 - z^3}}{\sqrt[3]{y - z}}$

Question 17

Rationalize the numerator

I. $\sqrt{\frac{2}{3}}$

II. $\sqrt[3]{\frac{7x}{3y}}$

Radicals Practice

Question 18

Rationalize the denominator

I. $\sqrt{\frac{2}{3}}$

II. $\sqrt[3]{\frac{7x}{3y}}$

Question 19

Solve

I. $7\sqrt{3} + 4\sqrt{3}$

II. $7\sqrt{3} - 4\sqrt{3}$

III. $5\sqrt{12} + 16\sqrt{27}$

IV. $-\sqrt{4x-4} + \sqrt{x-1}$

V. $\sqrt[3]{6x^4} - \sqrt[3]{48x}$

Radicals Practice

Question 20

Multiply

I. $-\sqrt{2} (5 + -\sqrt{2})$

II. $(-\sqrt{7} - \sqrt{2})(-\sqrt{2} + \sqrt{5})$

III. $(-\sqrt{3} - \sqrt{2})^2$

Question 21

Rationalize the numerator

I.
$$\frac{-\sqrt{5} + 1}{4}$$

II.
$$\frac{-\sqrt{x+y} - \sqrt{x}}{y}$$

Radicals Practice

Question 22

Rationalize the denominator

I.
$$\frac{6}{3 - \sqrt{2}}$$

Question 23

Solve

I.
$$\sqrt[10]{x} \sqrt[5]{x^2}$$

II.
$$-\sqrt{2x^3y^3} \sqrt[3]{4xy^2}$$

III.
$$\frac{\sqrt[3]{x^2}}{\sqrt[4]{x}}$$

IV.
$$\frac{\sqrt[4]{(5+3x)^3}}{\sqrt[3]{(5+3x)^2}}$$

Radicals Practice

V. $\sqrt[4]{x^2y} \left(\sqrt[3]{x^2y} - \sqrt[5]{x^2y^2} \right)$

VI. $\left(x - \sqrt[4]{y^3} \right) \left(3x - \sqrt[5]{y} \right)$

Question 24

Solve

I. $-\sqrt{5x+1} = 6$

II. $-\sqrt{x+5} - 4 = 1$

III. $6 - \sqrt{x} = x$

IV. $\sqrt[3]{x-2} = 3$

V. $2y^{1/2} - 13 = 7$

VI. $-\sqrt{x} = -2$

VII. $3 + \sqrt{5-x} = x$

Radicals Practice

Question 25

Solve

I. $-\sqrt{3x+4} = -\sqrt{4x+3}$

II. $3(4-x)^{1/4} = 6^{1/4}$

III. $-\sqrt{20-x} + 8 = -\sqrt{9-x} + 11$

IV. $-\sqrt{x+2} + \sqrt{3x+4} = 2$

Radicals Practice

Name: _____ **Key** _____

Date: _____

Question 1

Simplify

$$\begin{aligned} \text{I. } & \sqrt{16x^2} \\ & \downarrow \\ & \sqrt{(4x)^2} \\ & \downarrow \\ & |4x| \\ & \downarrow \\ & 4|x| \end{aligned}$$

$$\begin{aligned} \text{II. } & \sqrt{(-7x)^2} \\ & \downarrow \\ & |-7x| \\ & \downarrow \\ & 7|x| \end{aligned}$$

$$\begin{aligned} \text{III. } & \sqrt{(8-x)^2} \\ & \downarrow \\ & |8-x| \end{aligned}$$

$$\begin{aligned} \text{IV. } & \sqrt{x^2 + 16x + 64} \\ & \downarrow \\ & \sqrt{(x+8)^2} \\ & \downarrow \\ & |x+8| \end{aligned}$$

$$\begin{aligned} \text{V. } & \sqrt{x^{22}} \\ & \downarrow \\ & \sqrt{(x^{11})^2} \\ & \downarrow \\ & |x^{11}| \end{aligned}$$

Question 2

Simplify

$$\begin{aligned} \text{I. } & \sqrt{144} \\ & \downarrow \\ & 12 \end{aligned}$$

$$\begin{aligned} \text{II. } & \sqrt{\frac{36}{49}} \\ & \downarrow \\ & \frac{\sqrt{36}}{\sqrt{49}} \\ & \downarrow \\ & \frac{6}{7} \end{aligned}$$

$$\begin{aligned} \text{III. } & -\sqrt{100} \\ & \downarrow \\ & -10 \end{aligned}$$

$$\begin{aligned} \text{IV. } & \sqrt{0.0016} \\ & \downarrow \\ & 0.04 \end{aligned}$$

Question 3

Find the function value

$$\text{I. } f(x) = \sqrt{5x+10}$$

$$f(2) = \sqrt{5(2)+10} = \sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

$$f(3) = \sqrt{5(3)+10} = \sqrt{25} = 5$$

$$f(7) = \sqrt{5(7)+10} = \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Radicals Practice

Question 4

Simplify

$$\text{I. } \sqrt[3]{-1} \\ \downarrow \\ -1$$

$$\text{II. } -\sqrt[3]{64} \\ \downarrow \\ -4$$

$$\text{III. } \sqrt[3]{-64x^3} \\ \downarrow \\ -4x$$

Question 5

Simplify

$$\text{I. } -\sqrt[4]{256} \\ \downarrow \\ -4$$

$$\text{II. } \sqrt[6]{x^6} \\ \downarrow \\ |x|$$

$$\text{III. } \sqrt[5]{\frac{-1}{32}} \\ \downarrow \\ \sqrt[5]{\left(\frac{-1}{2}\right)^5} \\ \downarrow \\ \frac{-1}{2}$$

$$\text{IV. } \sqrt[4]{(6x)^4} \\ \downarrow \\ |6x| \\ \downarrow \\ 6|x|$$

$$\text{V. } \sqrt[12]{(-10)^{12}} \\ \downarrow \\ |-10| \\ \downarrow \\ 10$$

Question 6

Simplify

$$\text{I. } \sqrt{(x-2)^8} \\ \downarrow \\ \sqrt{((x-2)^4)^2} \\ \downarrow \\ (x-2)^4$$

$$\text{II. } \sqrt{(x+2)^{10}} \\ \downarrow \\ \sqrt{((x+2)^5)^2} \\ \downarrow \\ (x+2)^5$$

Question 7

Determine the domain

$$\text{I. } f(x) = \sqrt[5]{2x+7} \rightarrow \text{odd index} \rightarrow (-\infty, \infty)$$

$$\text{II. } f(x) = \sqrt{x+8} \rightarrow \text{even index} \rightarrow x+8 \geq 0 \rightarrow x \geq -8 \rightarrow [-8, \infty)$$

$$\text{III. } f(x) = \sqrt[3]{2x-6} \rightarrow \text{odd index} \rightarrow (-\infty, \infty)$$

$$\text{IV. } f(x) = -\sqrt[6]{5x+2} \rightarrow \text{even index} \rightarrow 5x+2 \geq 0 \rightarrow x \geq -2/5 \rightarrow [-2/5, \infty)$$

$$\text{V. } f(x) = 9 + \sqrt[6]{x^6} \rightarrow (-\infty, \infty)$$

Radicals Practice

Question 8

Convert to radical form

I. $x^{1/3}$
↓
 $\sqrt[3]{x}$

II. $36^{1/5}$
↓
 $\sqrt[5]{36}$

III. $x^{5/6}$
↓
 $\sqrt[6]{x^5}$

IV. $(9x^6)^{3/2}$
↓
 $\sqrt[2]{(9x^6)^3}$
↓
 $\sqrt[2]{9^3 \cdot x^{18}}$
↓
 $\sqrt[2]{9^3} \cdot \sqrt[2]{x^{18}}$
↓
 $(\sqrt[2]{9})^3 \cdot x^9$
↓
 $3^3 \cdot x^9$
↓
 $27x^9$

Question 9

Convert to exponent form

I. $\sqrt[3]{18}$
↓
 $18^{1/3}$

II. $\sqrt[5]{x^4}$
↓
 $x^{4/5}$

III. $\sqrt{24}$
↓
 $24^{1/2}$

IV. $\sqrt[7]{x^3y^2z^3}$
↓
 $(x^3y^2z^3)^{1/7}$

V. $(\sqrt[6]{2x^5y})^7$
↓
 $(2x^5y)^{7/6}$

VI. $\frac{3x}{\sqrt[5]{y^2}}$
↓
 $\frac{3x}{y^{2/5}}$

Question 10

Convert to radical form

I. $6^{-1/3}$
↓
 $\frac{1}{6^{1/3}}$
↓
 $\frac{1}{\sqrt[3]{6}}$

II. $(\frac{1}{15})^{-3/4}$
↓
 $(\frac{15}{1})^{3/4}$
↓
 $\sqrt[4]{15^3}$

III. $\frac{8x}{y^{-3/5}}$
↓
 $8xy^{3/5}$
↓
 $8x\sqrt[5]{y^3}$

IV. $(\frac{2xy}{3z})^{-5/6}$
↓
 $(\frac{3z}{2xy})^{5/6}$
↓
 $\sqrt[6]{(\frac{3z}{2xy})^5}$

Radicals Practice

Question 11

Simplify

I. $\sqrt[9]{y^3} \rightarrow y^{3/9} \rightarrow y^{1/3} \rightarrow \sqrt[3]{y}$

II. $(\sqrt[7]{xy})^{14} \rightarrow (xy)^{14/7} \rightarrow (xy)^{2/1} \rightarrow (xy)^2 \rightarrow x^2y^2$

III. $\sqrt{\sqrt[5]{x}} \rightarrow (x^{1/5})^{1/2} \rightarrow x^{1/10} \rightarrow \sqrt[10]{x}$

IV. $\sqrt[3]{\sqrt[4]{xy}} \rightarrow ((xy)^{1/4})^{1/3} \rightarrow (xy)^{1/12} \rightarrow \sqrt[12]{xy}$

V. $\sqrt[4]{(xy)^{12}} \rightarrow (xy)^{12/4} \rightarrow (xy)^3 \rightarrow x^3y^3$

Question 12

Multiply

I. $\sqrt{10} \sqrt{3} \rightarrow \sqrt{10 \cdot 3} \rightarrow \sqrt{30}$

II. $\sqrt[3]{2} \sqrt[3]{3} \rightarrow \sqrt[3]{2 \cdot 3} \rightarrow \sqrt[3]{6}$

III. $\sqrt{5x} \sqrt{6y} \rightarrow \sqrt{5x \cdot 6y} \rightarrow \sqrt{30xy}$

IV. $\sqrt[5]{9x^2} \sqrt[5]{2x} \rightarrow \sqrt[5]{9x^2 \cdot 2x} \rightarrow \sqrt[5]{18x^3}$

V. $\sqrt[3]{9x^2} \sqrt[3]{2y} \rightarrow \sqrt[3]{9x^2 \cdot 2y} \rightarrow \sqrt[3]{18x^2y}$

VI. $\sqrt[4]{y-1} \sqrt[4]{y^2+y+1} \rightarrow \sqrt[4]{(y-1)(y^2+y+1)} \rightarrow \sqrt[4]{y^3-1}$

VII. $\sqrt{\frac{7x}{6}} \sqrt{\frac{5}{y}} \rightarrow \sqrt{\frac{7x}{6} \cdot \frac{5}{y}} \rightarrow \sqrt{\frac{35x}{6y}}$

Radicals Practice

Question 13

Simplify

$$I. \quad \sqrt{12} \rightarrow \sqrt{4 \cdot 3} \rightarrow \sqrt{4} \sqrt{3} \rightarrow 2 \sqrt{3}$$

$$II. \quad \sqrt{300} \rightarrow \sqrt{100 \cdot 3} \rightarrow \sqrt{100} \sqrt{3} \rightarrow 10 \sqrt{3}$$

$$III. \quad \sqrt{36x^4y} \rightarrow \sqrt{36} \sqrt{(x^2)^2} \sqrt{y} \rightarrow 6x^2 \sqrt{y}$$

$$IV. \quad \sqrt[3]{-32x^6} \rightarrow \sqrt[3]{-8 \cdot 4 \cdot (x^2)^3} \rightarrow -2x^2 \sqrt[3]{4}$$

$$V. \quad \sqrt[5]{-32x^7y^{11}} \rightarrow \sqrt[5]{-32 \cdot x^5 \cdot x^2 \cdot y^5 \cdot y^5 \cdot y} \rightarrow -2xy^2 \sqrt[5]{x^2y}$$

$$VI. \quad \sqrt{x^5y^6z^{10}} \rightarrow \sqrt{x^2 \cdot x^2 \cdot x \cdot y^2 \cdot y^2 \cdot y^2 \cdot z^2 \cdot z^2 \cdot z^2 \cdot z^2} \rightarrow x^2y^3z^5 \sqrt{x}$$

Question 14

Multiply

$$I. \quad \sqrt{10} \sqrt{5} \rightarrow \sqrt{50} \rightarrow \sqrt{25 \cdot 2} \rightarrow \sqrt{25} \sqrt{2} \rightarrow 5 \sqrt{2}$$

$$II. \quad \sqrt{24x^5} \sqrt{24x^5} \rightarrow \sqrt{24 \cdot 24 \cdot x^5 \cdot x^5} \rightarrow \sqrt{(24)^2 x^{10}} \rightarrow 24x^5$$

$$III. \quad 3 \sqrt{5x^7} \cdot 2 \sqrt{15x^3} \rightarrow 6 \sqrt{75x^{10}} \rightarrow 6 \sqrt{25 \cdot 3 \cdot x^{10}} \rightarrow 6 \cdot 5 \cdot x^5 \sqrt{3} \rightarrow 30x^5 \sqrt{3}$$

$$IV. \quad \sqrt[5]{x^3(y-z)^4} \sqrt[5]{x^7(y-z)^4} \rightarrow \sqrt[5]{x^{10}(y-z)^8} \rightarrow \sqrt[5]{x^{10}(y-z)^5(y-z)^3} \rightarrow x^2(y-z) \sqrt[5]{(y-z)^3}$$

Radicals Practice

Question 15

Simplify

I. $\sqrt{\frac{49}{100}} \rightarrow \frac{\sqrt{49}}{\sqrt{100}} \rightarrow \frac{7}{10}$ II. $\sqrt{\frac{25x^5}{y^6}} \rightarrow \frac{\sqrt{25x^5}}{\sqrt{y^6}} \rightarrow \frac{\sqrt{25 \cdot x^2 \cdot x^2 \cdot x}}{\sqrt{y^6}} \rightarrow \frac{5x^2\sqrt{x}}{y^3}$

III. $\sqrt[3]{\frac{27x^4}{8y^3}} \rightarrow \frac{\sqrt[3]{27x^4}}{\sqrt[3]{8y^3}} \rightarrow \frac{\sqrt[3]{27 \cdot x^3 \cdot x}}{\sqrt[3]{8y^3}} \rightarrow \frac{3x\sqrt[3]{x}}{2y}$

Question 16

Divide

I. $\frac{\sqrt{18x}}{\sqrt{2x}} \rightarrow \sqrt{\frac{18x}{2x}} \rightarrow \sqrt{9} \rightarrow 3$ II. $\frac{\sqrt[3]{189y^5z^7}}{\sqrt[3]{7y^2z^2}} \rightarrow \sqrt[3]{\frac{189y^5z^7}{7y^2z^2}} \rightarrow \sqrt[3]{27y^3z^5} \rightarrow 3yz\sqrt[3]{z^2}$

III. $\frac{\sqrt[3]{y^3 - z^3}}{\sqrt[3]{y - z}} \rightarrow \frac{\sqrt[3]{(y - z)(y^2 + yz + z^2)}}{\sqrt[3]{y - z}} \rightarrow \sqrt[3]{\frac{(y - z)(y^2 + yz + z^2)}{y - z}} \rightarrow \sqrt[3]{y^2 + yz + z^2}$

Question 17

Rationalize the numerator

I. $\sqrt{\frac{2}{3}} \rightarrow \sqrt{\frac{2}{3} \cdot \frac{2}{2}} \rightarrow \sqrt{\frac{4}{6}} \rightarrow \frac{\sqrt{4}}{\sqrt{6}} \rightarrow \frac{2}{\sqrt{6}}$

II. $\sqrt[3]{\frac{7x}{3y}} \rightarrow \sqrt[3]{\frac{7x}{3y} \cdot \frac{49x^2}{49x^2}} \rightarrow \sqrt[3]{\frac{343x^3}{147x^2y}} \rightarrow \frac{7x}{\sqrt[3]{147x^2y}}$

Radicals Practice

Question 18

Rationalize the denominator

$$I. \quad \sqrt{\frac{2}{3}} \rightarrow \sqrt{\frac{2}{3} \cdot \frac{3}{3}} \rightarrow \sqrt{\frac{6}{9}} \rightarrow \frac{\sqrt{6}}{\sqrt{9}} \rightarrow \frac{\sqrt{6}}{3}$$

$$II. \quad \sqrt[3]{\frac{7x}{3y}} \rightarrow \sqrt[3]{\frac{7x}{3y} \cdot \frac{9y^2}{9y^2}} \rightarrow \sqrt[3]{\frac{63xy^2}{27y^3}} \rightarrow \frac{\sqrt[3]{63xy^2}}{3y}$$

Question 19

Solve

$$I. \quad 7\sqrt{3} + 4\sqrt{3} \rightarrow (7+4)\sqrt{3} \rightarrow 11\sqrt{3}$$

$$II. \quad 7\sqrt{3} - 4\sqrt{3} \rightarrow (7-4)\sqrt{3} \rightarrow 3\sqrt{3}$$

$$III. \quad 5\sqrt{12} + 16\sqrt{27} \rightarrow 5\sqrt{4 \cdot 3} + 16\sqrt{9 \cdot 3} \rightarrow 10\sqrt{3} + 48\sqrt{3}$$

$$\downarrow$$

$$58\sqrt{3} \leftarrow (10+48)\sqrt{3}$$

$$IV. \quad \sqrt{4x-4} + \sqrt{x-1} \rightarrow \sqrt{4(x-1)} + \sqrt{x-1} \rightarrow 2\sqrt{x-1} + \sqrt{x-1}$$

$$\downarrow$$

$$3\sqrt{x-1} \leftarrow (2+1)\sqrt{x-1}$$

$$V. \quad \sqrt[3]{6x^4} - \sqrt[3]{48x} \rightarrow \sqrt[3]{6 \cdot x^3 \cdot x} - \sqrt[3]{8 \cdot 6 \cdot x} \rightarrow x\sqrt[3]{6x} - 2\sqrt[3]{6x}$$

$$\downarrow$$

$$(x-2)\sqrt[3]{6x} \leftarrow (x-2)\sqrt[3]{6x}$$

Radicals Practice

Question 20

Multiply

$$I. \quad -\sqrt{2} (5 + \sqrt{2}) \rightarrow -\sqrt{2} \cdot 5 + \sqrt{2} \cdot \sqrt{2} \rightarrow 5\sqrt{2} + 2$$

$$II. \quad (\sqrt{7} - \sqrt{2})(\sqrt{2} + \sqrt{5}) \rightarrow \sqrt{14} + \sqrt{35} - 2 - \sqrt{10}$$

$$III. \quad (\sqrt{3} - \sqrt{2})^2 \rightarrow (\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2}) \rightarrow 3 - \sqrt{6} - \sqrt{6} + 2$$

$$\downarrow$$

$$5 - 2\sqrt{6}$$

Question 21

Rationalize the numerator

$$I. \quad \frac{\sqrt{5} + 1}{4} \rightarrow \frac{\sqrt{5} + 1}{4} \cdot \frac{\sqrt{5} - 1}{\sqrt{5} - 1} \rightarrow \frac{5 - 1}{4(\sqrt{5} - 1)}$$

$$\downarrow$$

$$\frac{1}{\sqrt{5} - 1} \leftarrow \frac{4}{4(\sqrt{5} - 1)}$$

$$II. \quad \frac{\sqrt{x+y} - \sqrt{x}}{y} \rightarrow \frac{\sqrt{x+y} - \sqrt{x}}{y} \cdot \frac{\sqrt{x+y} + \sqrt{x}}{\sqrt{x+y} + \sqrt{x}} \rightarrow \frac{x+y-x}{y(\sqrt{x+y} + \sqrt{x})}$$

$$\downarrow$$

$$\frac{1}{\sqrt{x+y} + \sqrt{x}} \leftarrow \frac{y}{y(\sqrt{x+y} + \sqrt{x})}$$

Radicals Practice

Question 22

Rationalize the denominator

$$\begin{aligned}
 \text{I. } \frac{6}{3 - \sqrt{2}} &\rightarrow \frac{6}{3 - \sqrt{2}} \cdot \frac{3 + \sqrt{2}}{3 + \sqrt{2}} \rightarrow \frac{18 + 6\sqrt{2}}{9 - 2} \\
 &\quad \downarrow \\
 &\frac{18 + 6\sqrt{2}}{7}
 \end{aligned}$$

Question 23

Solve

$$\text{I. } \sqrt[10]{x} \sqrt[5]{x^2} \rightarrow x^{1/10} \cdot x^{2/5} \rightarrow x^{1/2} \rightarrow \sqrt{x}$$

$$\begin{aligned}
 \text{II. } \sqrt{2x^3y^3} \sqrt[3]{4xy^2} &\rightarrow (2x^3y^3)^{1/2} \cdot (4xy^2)^{1/3} \rightarrow (2x^3y^3)^{3/6} \cdot (4xy^2)^{2/6} \rightarrow ((2x^3y^3)^3 \cdot (4xy^2)^2)^{1/6} \\
 &\quad \downarrow \\
 &\sqrt[6]{64x^9y^9 \cdot 16x^2y^4} \leftarrow \sqrt[6]{128x^{11}y^{13}} \leftarrow \sqrt[6]{8x^9y^9 \cdot 16x^2y^4} \leftarrow ((8x^9y^9) \cdot (16x^2y^4))^{1/6} \\
 &\quad \downarrow \\
 &2xy^2 \sqrt[6]{2x^5y}
 \end{aligned}$$

$$\text{III. } \frac{\sqrt[3]{x^2}}{\sqrt[4]{x}} \rightarrow x^{2/3} / x^{1/4} \rightarrow x^{5/12} \rightarrow \sqrt[12]{x^5}$$

$$\text{IV. } \frac{\sqrt[4]{(5+3x)^3}}{\sqrt[3]{(5+3x)^2}} \rightarrow (5+3x)^{3/4} / (5+3x)^{2/3} \rightarrow (5+3x)^{1/12} \rightarrow \sqrt[12]{5+3x}$$

Radicals Practice

$$\begin{aligned}
 \text{V. } \sqrt[4]{x^2y} \left(\sqrt[3]{x^2y} - \sqrt[5]{x^2y^2} \right) &\rightarrow (x^2y)^{1/4} \left((x^2y)^{1/3} - (x^2y^2)^{1/5} \right) \rightarrow (x^{2/4}y^{1/4}) (x^{2/3}y^{1/3} - x^{2/5}y^{2/5}) \\
 &\downarrow \\
 x \cdot x^{2/12}y^{7/12} - x^{18/20}y^{13/20} &\leftarrow x^{14/12}y^{7/12} - x^{18/20}y^{13/20} \leftarrow x^{7/6}y^{7/12} - x^{9/10}y^{13/20} \\
 &\downarrow \\
 x(x^{2/12}y^{7/12} - x^{18/20}y^{13/20}) &\rightarrow x \sqrt[12]{x^2y^7} - \sqrt[20]{x^{18}y^{13}}
 \end{aligned}$$

$$\begin{aligned}
 \text{VI. } \left(x - \sqrt[4]{y^3} \right) \left(3x - \sqrt[5]{y} \right) &\rightarrow (x - y^{3/4})(3x - y^{1/5}) \rightarrow 3x^2 - xy^{1/5} - 3xy^{3/4} + y^{19/20} \\
 &\downarrow \\
 3x^2 - x \sqrt[5]{y} - 3x \sqrt[4]{y^3} + \sqrt[20]{y^{19}} &
 \end{aligned}$$

Question 24

Solve

$$\text{I. } \sqrt{5x+1} = 6 \rightarrow \left(\sqrt{5x+1} \right)^2 = 6^2 \rightarrow 5x+1=36 \rightarrow x=7$$

$$\text{II. } \sqrt{x+5} - 4 = 1 \rightarrow \left(\sqrt{x+5} \right)^2 = 5^2 \rightarrow x+5=25 \rightarrow x=20$$

$$\text{III. } 6\sqrt{x} = x \rightarrow \left(6\sqrt{x} \right)^2 = x^2 \rightarrow 36x = x^2 \rightarrow 0 = x^2 - 36x \rightarrow 0 = x(x-36) \rightarrow \begin{matrix} x=0 \\ x=36 \end{matrix}$$

$$\text{IV. } \sqrt[3]{x-2} = 3 \rightarrow \left(\sqrt[3]{x-2} \right)^3 = 3^3 \rightarrow x-2=27 \rightarrow x=29$$

$$\text{V. } 2y^{1/2} - 13 = 7 \rightarrow 2y^{1/2} = 20 \rightarrow y^{1/2} = 10 \rightarrow (y^{1/2})^2 = 10^2 \rightarrow y=100$$

$$\text{VI. } \sqrt{x} = -2 \rightarrow \text{no solution}$$

$$\text{VII. } 3 + \sqrt{5-x} = x \rightarrow \sqrt{5-x} = x-3 \rightarrow \left(\sqrt{5-x} \right)^2 = (x-3)^2 \rightarrow 5-x = x^2 - 6x + 9$$

$$\begin{aligned}
 &\downarrow \\
 x=4 &\leftarrow 0 = x^2 - 5x + 4 \leftarrow 5 = x^2 - 5x + 9 \\
 \text{not true } \cancel{x=1} &
 \end{aligned}$$

Radicals Practice

Question 25

Solve

$$\text{I. } \sqrt{3x+4} = \sqrt{4x+3} \rightarrow (\sqrt{3x+4})^2 = (\sqrt{4x+3})^2 \rightarrow 3x+4 = 4x+3$$
$$\downarrow$$
$$x=1$$

$$\text{II. } 3(4-x)^{1/4} = 6^{1/4} \rightarrow (3(4-x)^{1/4})^4 = (6^{1/4})^4 \rightarrow 81(4-x) = 6 \rightarrow 324 - 81x = 6 \rightarrow x = 106/27$$

$$\text{III. } \sqrt{20-x} + 8 = \sqrt{9-x} + 11 \rightarrow \sqrt{20-x} = \sqrt{9-x} + 3$$
$$\downarrow$$
$$(\sqrt{20-x})^2 = (\sqrt{9-x} + 3)^2$$
$$\downarrow$$
$$20-x = -x + 18 + 6\sqrt{-x+9}$$
$$\downarrow$$
$$(2)^2 = (6\sqrt{-x+9})^2 \leftarrow 2 = 6\sqrt{-x+9}$$
$$\downarrow$$
$$4 = 36(-x+9) \rightarrow 4 = -36x + 324 \rightarrow 80/9 = x$$

$$\text{IV. } \sqrt{x+2} + \sqrt{3x+4} = 2 \rightarrow \sqrt{x+2} = 2 - \sqrt{3x+4} \rightarrow (\sqrt{x+2})^2 = (2 - \sqrt{3x+4})^2$$
$$\downarrow$$
$$(-2x-6)^2 = (-4\sqrt{3x+4})^2 \leftarrow -2x-6 = -4\sqrt{3x+4} \leftarrow x+2 = 3x+8 - 4\sqrt{3x+4}$$
$$\downarrow$$
$$4x^2 + 24x + 36 = 48x + 64 \rightarrow 4x^2 - 24x - 28 = 0$$
$$\downarrow$$

not true ~~$x=7$~~

$$x = -1$$