

Differentiation Rules Practice

Name: _____

Date: _____

Question 1

Solve for the second derivative

1. $y = -x^2 + 3$

Question 2

Solve for the derivative

1. $(2x + 3)(5x^2 - 4x)$

Differentiation Rules Practice

II. $\frac{2x+5}{3x-2}$

III. x^3e^x

Differentiation Rules Practice

IV. $2e^{-x} + e^{3x}$

V. x^3e^x (Power / Natural Exponential)

Differentiation Rules Practice

VI. xe^{-x}

VII. $\sqrt[7]{x^2} - x^e$

Differentiation Rules Practice

VIII. $\frac{e^x}{x}$

Differentiation Rules Practice

IX. $e^x \left(\frac{1}{x^2} + x^{-\pi/2} \right)$

Differentiation Rules Practice

Name: _____ **Key** _____

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Question 1

Solve for the second derivative

i. $y = -x^2 + 3$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{-(x+h)^2 + 3 - (-x^2 + 3)}{h} &\longrightarrow \frac{-2hx - h^2}{h} \longrightarrow \frac{\cancel{h}(-2x - h)}{\cancel{h}} \longrightarrow -2x - h \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad -2x - (0) \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad -2x \end{aligned}$$

$$\lim_{h \rightarrow 0} \frac{-2(x+h) - (-2(x))}{h} \longrightarrow \frac{\cancel{-2}h}{\cancel{h}} \longrightarrow -2$$

Question 2

Solve for the derivative

i. $(2x+3)(5x^2-4x)$

$$(2x+3)(5x^2-4x) \longrightarrow 10x^3 + 7x^2 - 12x$$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(10(x+h)^3 + 7(x+h)^2 - 12(x+h)) - (10x^3 + 7x^2 - 12x)}{h} &\longrightarrow \frac{30hx^2 + 30h^2x + 10h^3 + 14hx + 7h^2 - 12h}{h} \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad \frac{\cancel{h}(30x^2 + 30hx + 10h^2 + 14x + 7h - 12)}{\cancel{h}} \\ &\qquad\qquad\qquad \longleftarrow 30x^2 + 30hx + 10h^2 + 14x + 7h - 12 \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad 30x^2 + 30(0)x + 10(0)^2 + 14x + 7(0) - 12 \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad 30x^2 + 14x - 12 \end{aligned}$$

Differentiation Rules Practice

II. $\frac{2x+5}{3x-2}$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\left(\frac{2(x+h)+5}{3(x+h)-2}\right) - \left(\frac{2x+5}{3x-2}\right)}{h} &\longrightarrow \frac{-19h}{(3x+3h-2)(3x-2)} \longrightarrow \frac{-19\cancel{h}}{(3x+3h-2)(3x-2)} \cdot \frac{1}{\cancel{h}} \\ &\downarrow \\ \frac{-19}{(3x-2)^2} &\longleftarrow \frac{-19}{(3x-2)(3x-2)} \longleftarrow \frac{-19}{(3x+3(0)-2)(3x-2)} \longleftarrow \frac{-19}{(3x+3h-2)(3x-2)} \end{aligned}$$

III. x^3e^x

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(x+h)^3 - (x)^3}{h} &\longrightarrow \frac{3hx^2 + 3h^2x + h^3}{h} \longrightarrow \frac{\cancel{h}(3x^2 + 3hx + h^2)}{\cancel{h}} \longrightarrow 3x^2 + 3hx + h^2 \\ &\downarrow \\ 3x^2 &\longleftarrow 3x^2 + 3(0)x + (0)^2 \longleftarrow 3x^2 + 3(0)x + (0)^2 \end{aligned}$$

$$\frac{d}{dx} (e^x) \longrightarrow e^x$$

Natural Exponential Function

$$(x^3 \cdot e^x) + (e^x \cdot 3x^2) \longrightarrow x^3e^x + 3x^2e^x \longrightarrow e^x(x^3 + 3x^2)$$

Product Rule

Differentiation Rules Practice

IV. $2e^{-x} + e^{3x}$

Product Rule (Special Natural Exponential)

$$2 \frac{d}{dx} (e^{-x}) \longrightarrow (-e^{-x}) \longrightarrow (2)(-e^{-x}) \longrightarrow -2e^{-x}$$

Constant Multiple

$$\frac{d}{dx} e^{3x} \longrightarrow 3e^{3x}$$

Product Rule (Special Natural Exponential)

$$-2e^{-x} + 3e^{3x}$$

Sum Rule

V. x^3e^x (Power / Natural Exponential)

$$\frac{d}{dx} x^3 \longrightarrow 3x^2$$

Power Rule

$$\frac{d}{dx} e^x \longrightarrow e^x$$

Natural Exponential Function

$$e^x \cdot 3x^2 + e^x \cdot x^3 \longrightarrow e^x(x^3 + 3x^2)$$

Product Rule

Differentiation Rules Practice

VI. $x e^{-x}$

$$\lim_{h \rightarrow 0} \frac{(x+h) - (x)}{h} \longrightarrow \frac{h}{h} \longrightarrow 1$$

$$\frac{d}{dx} (e^{-x}) \longrightarrow -e^{-x}$$

Product Rule (Special Natural Exponential)

$$x \cdot -e^{-x} + e^{-x} \cdot 1 \longrightarrow x \cdot -e^{-x} + e^{-x} \longrightarrow -x e^{-x} + e^{-x} \longrightarrow e^{-x}(-x + 1)$$

Product Rule

VII. $\sqrt[7]{x^2} - x^e$ $\longrightarrow x^{2/7} - x^e \longrightarrow x^{2/7} + -x^e$

$$\frac{d}{dx} (x^{2/7}) \longrightarrow (2/7)x^{2/7-1} \longrightarrow (2/7)x^{-5/7}$$

Power Rule

$$\frac{d}{dx} (-x^e) \longrightarrow -e x^{e-1}$$

Power Rule

$$(2/7)x^{-5/7} + -e x^{e-1} \longrightarrow \frac{2}{7x^{5/7}} + -e x^{e-1}$$

Sum Rule

Differentiation Rules Practice

VIII. $\frac{e^x}{x}$

$$\lim_{h \rightarrow 0} \frac{(x+h) - (x)}{h} \longrightarrow \frac{h}{h} \longrightarrow 1$$

$$\frac{d}{dx} e^x \longrightarrow e^x$$

Natural Exponential Function

$$\frac{x \cdot e^x - e^x \cdot 1}{x^2} \longrightarrow \frac{x e^x - e^x}{x^2} \longrightarrow \frac{e^x(x-1)}{x^2}$$

Quotient Rule

Differentiation Rules Practice

IX. $e^x \left(\frac{1}{x^2} + x^{-\pi/2} \right)$

$$\frac{1}{x^2} \longrightarrow x^{-2} \longrightarrow \frac{d}{dx} (x^{-2}) \longrightarrow (-2x^{-3})$$

Power Rule

$$x^{-\pi/2} \longrightarrow \frac{d}{dx} (x^{-\pi/2}) \longrightarrow \left((-\pi/2) x^{-(\pi+2)/2} \right)$$

Power Rule

$$-2x^{-3} + (-\pi/2)x^{-(\pi+2)/2} \longrightarrow \frac{-2}{x^3} - \frac{\pi}{2x^{(\pi+2)/2}}$$

Sum Rule

$$\frac{d}{dx} e^x \longrightarrow e^x$$

Natural Exponential Function

$$e^x \cdot \left(\frac{-2}{x^3} - \frac{\pi}{2x^{(\pi+2)/2}} \right) + e^x \cdot \left(\frac{1}{x^2} + x^{-\pi/2} \right) \longrightarrow e^x \left(\frac{-2}{x^3} - \frac{\pi}{2x^{(\pi+2)/2}} + \frac{1}{x^2} + x^{-\pi/2} \right)$$

Product Rule

$$\downarrow$$
$$e^x \left(\frac{x-2}{x^3} + \frac{2x-\pi}{2x^{(\pi+2)/2}} \right)$$