Chain Rule

Name: ____________________ Date: ____________________

Question 1

Find the derivative

I. $y = 6u - 9$ $u = \frac{1}{2}x^4$

II. $y = \tan u$ $u = \pi x^2$

Chain Rule

III. $y = 2u^3$ $u = 8x - 1$

Question 2

Find the derivative

I. $y = (2x + 1)^5$

Chain Rule

II. $y = \tan^3 x$

III. $y = \sqrt{3 - x} \longrightarrow (3 - x)^{1/2}$

Chain Rule

IV. $y = xe^{-x} + e^{x^3}$

V. $y = e^{-5x}$

VI. $y = e^{2x/3}$

VII. $y = e^{5 - 7x}$

VIII. $y = e^{(4\sqrt{x} + x^2)}$

IX. $y = e^{\cos^2(\pi x - 1)}$

Chain Rule

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Question 1

Find the derivative

I. $y = 6u - 9 \quad u = \frac{1}{2}x^4$

$f'(u) / f'(g(x))$:

$$\lim_{h \to 0} \frac{(6(x+h)-9)-(6(x)-9)}{h} \longrightarrow \frac{6\cancel{h}}{\cancel{h}} \longrightarrow 6$$

$g'(x)$:

$$\frac{1}{2}\frac{dy}{dx}x^4 \longrightarrow \frac{1}{2}\cdot 4x^3 \longrightarrow 2x^3$$

$$f'(g(x))g'(x) = 6 \cdot 2x^3 \longrightarrow 12x^3$$

II. $y = \tan u \quad u = \pi x^2$

$f'(u)$:

$$\frac{dy}{dx}\tan u \longrightarrow \sec^2 u$$

$f'(g(x))$:

$$\sec^2(\pi x^2)$$

$g'(x)$:

$$\pi \frac{dy}{dx}x^2 \longrightarrow \pi \cdot 2x \longrightarrow 2\pi x$$

$$f'(g(x))g'(x) = \sec^2(\pi x^2) \cdot 2\pi x \longrightarrow 2\pi x \sec^2(\pi x^2)$$

Chain Rule

III. $y = 2u^3$ $\quad u = 8x - 1$

f'(u) :

$$2\frac{dy}{dx}u^3 \longrightarrow 2 \cdot 3u^2 \longrightarrow 6u^2$$

f'(g(x)) :

$6(8x - 1)^2$

g'(x) :

$$\lim_{h \to 0} \frac{(8(x+h) - 1) - (8(x) - 1)}{h} \longrightarrow \frac{8\cancel{h}}{\cancel{h}} \longrightarrow 8$$

$$f'(g(x))g'(x) = 6(8x - 1)^2 \cdot 8 \longrightarrow 48(8x - 1)^2$$

Question 2

Find the derivative

I. $y = (2x + 1)^5$

$y = u^5$ $\quad u = (2x + 1)$

f'(u) :

$$\frac{dy}{dx}u^5 \longrightarrow 5u^4$$

$$f'(g(x))g'(x) = 5(2x + 1)^4 \cdot 2 \longrightarrow 10(2x + 1)^4$$

f'(g(x)) :

$5(2x + 1)^4$

g'(x) :

$$\lim_{h \to 0} \frac{(2(x+h) + 1) - (2(x) + 1)}{h} \longrightarrow \frac{2\cancel{h}}{\cancel{h}} \longrightarrow 2$$

Chain Rule

II. $y = \tan^3 x$

$y = u^3 \qquad u = \tan x$

$f'(u)$:

$\frac{dy}{dx} u^3 \longrightarrow 3u^2$

$f'(g(x))g'(x) = 3(\tan x)^2 \cdot \sec^2 x \longrightarrow 3\tan^2 x \sec^2 x$

$f'(g(x))$:

$3(\tan x)^2$

$g'(x)$:

$\frac{dy}{dx} \tan x \longrightarrow \sec^2 x$

III. $y = \sqrt{3 - x} \longrightarrow (3 - x)^{1/2}$

$y = u^{1/2} \qquad u = 3 - x$

$f'(u)$:

$\frac{dy}{dx} u^{1/2} \longrightarrow (1/2)u^{-1/2}$

$f'(g(x))g'(x) = \frac{1}{2(3 - x)^{1/2}} \cdot -1 \longrightarrow \frac{-1}{2(3 - x)^{1/2}}$

$f'(g(x))$:

$(1/2)(3 - x)^{-1/2} \longrightarrow \frac{1}{2(3 - x)^{1/2}}$

$g'(x)$:

$\lim_{h \to 0} \frac{(3 - (x + h)) - (3 - (x))}{h} \longrightarrow \frac{-h}{h} \longrightarrow -1$

Chain Rule

IV. $y = xe^{-x} + e^{x^3}$

$\frac{dy}{dx}\ xe^{-x} \longrightarrow (1 - x)e^{-x}$

$\frac{dy}{dx}\ e^{x^3} \longrightarrow 3x^2e^{x^3}$

$(1 - x)e^{-x} + 3x^2e^{x^3}$

V. $y = e^{-5x}$

$\frac{dy}{dx}\ e^{-5x} \longrightarrow -5e^{-5x}$

VI. $y = e^{2x/3}$

$\frac{dy}{dx}\ e^{2x/3} \longrightarrow (2/3)e^{2x/3}$

VII. $y = e^{5-7x}$

$\frac{dy}{dx}\ e^{5-7x} \longrightarrow -7e^{5-7x}$

VIII. $y = e^{(4\sqrt{x} + x^2)}$

$\frac{dy}{dx}\ e^{(4\sqrt{x} + x^2)} \longrightarrow (2x^{-1/2} + 2x)e^{(4\sqrt{x} + x^2)}$

$4 \cdot (1/2)x^{-1/2} + 2x \longrightarrow 2x^{-1/2} + 2x$

IX. $y = e^{\cos^2(\pi x - 1)}$

$\frac{dy}{dx}\ \cos^2(\pi x - 1)$

$y = u^2$ $\quad u = \cos x$

$f'(u)$:

$\frac{dy}{dx}\ u^2 \longrightarrow 2u$

$g'(x)$:

$\frac{dy}{dx}\ \cos x \longrightarrow -\sin x$

$f'(g(x))$:

$2\cos x$

$f'(g(x))g'(x) = 2\cos x \cdot -\sin x$

$\downarrow$

$-2\cos x \sin x$

$\downarrow$

$-2\cos(\pi x - 1)\sin(\pi x - 1)$

Chain Rule

$$\lim_{h \to 0} \frac{(\pi(x + h) - 1) - (\pi(x) - 1)}{h} \longrightarrow \frac{\pi \cancel{h}}{\cancel{h}} \longrightarrow \pi$$

$$-2\cos(\pi x - 1)\sin(\pi x - 1) \cdot \pi \cdot e^{\cos^2(\pi x - 1)} \longrightarrow -2\pi\cos(\pi x - 1)\sin(\pi x - 1)e^{\cos^2(\pi x - 1)}$$