Rational Expression Operations

Name: _____ Date: _____

Question 1

Simplify

I. $\frac{8x^3}{5x} \cdot \frac{3}{4x}$

II. $\frac{3y^2z}{2} \cdot \frac{4}{yz^3}$

III. $\frac{x^2 - y^2}{4x + 4y} \cdot \frac{x + y}{x - y}$

Rational Expression Operations

IV. $\frac{x^2 - 6x + 5}{x + 6} \cdot \frac{x - 6}{x^2 + 36}$

V. $\frac{x^2 + 2x - 3}{x^2 + 4x - 5} \cdot \frac{x^2 + 3x - 10}{x^2 + 5x + 6}$

VI. $\frac{y^2 + 4y + 4}{(y - 1)^2} \cdot \frac{y^2 - 2y + 1}{(y + 2)^2}$

Rational Expression Operations

VII. $\frac{y}{4} \div \frac{5}{y}$

VIII. $\frac{4x-8}{x+2} \div \frac{x-2}{x^2-4}$

IX. $\frac{x^2-14x+49}{2x^2-3x-14} \div \frac{3x^2-20x-7}{x^2-6x-16}$

Rational Expression Operations

X. $\frac{3}{x} + \frac{5}{x}$

XI. $\frac{y}{12} + \frac{2y+5}{12}$

XII. $\frac{x^2-5x}{x^2+6x+9} + \frac{4x-12}{x^2+6x+9}$

XIII. $\frac{9}{2y+3} - \frac{5}{2y+3}$

Rational Expression Operations

XIV. $\frac{2x^2+3x}{x^2-7x+12} - \frac{x^2+4x+6}{x^2-7x+12}$

XV. $\frac{3}{x^2} + \frac{4}{x}$

XVI. $\frac{3}{x+2} + \frac{7}{(x+2)^2}$

XVII. $\frac{x}{x-1} - \frac{x-1}{x}$

Rational Expression Operations

XVIII. $\frac{x}{x^2+2x+1} + \frac{1}{x^2+5x+4}$

XIX. $\frac{7}{x^2+25x+24} - \frac{0}{x^2+15x+56}$

Rational Expression Operations

XX. $2 + \frac{1}{5 - x}$

XXI. $\frac{5y}{4} + \frac{y - 2}{-4}$

XXII. $\frac{x - 5}{x^2 - 64} - \frac{5 - x}{64 - x^2}$

Rational Expression Operations

XXIII. $\frac{3y+2}{3y+6} + \frac{y}{4-y^2}$

XXIV. $\frac{\frac{2}{x^2y} + \frac{3}{xy^2}}{\frac{3}{xy^2} + \frac{2}{x^2y}}$

Rational Expression Operations

XXV. $$\frac{\frac{z}{z^2+3z-4} - \frac{1}{z^2+3z-4}}{\frac{z}{z^2+6z+8} + \frac{3}{z^2+6z+8}}$$

XXVI. $$\frac{y-3-\frac{2}{y}}{y-4-\frac{3}{y}}$$

Rational Expression Operations

XXVII. $\dfrac{x + x^{-1}}{x - x^{-1}}$

Rational Expression Operations

Question 2

Solve

I. $\frac{5}{8} - \frac{3}{5} = \frac{x}{10}$

II. $\frac{y}{6} - \frac{6}{y} = 0$

III. $\frac{x-4}{x+6} = \frac{1}{3}$

IV. $\frac{y+2}{5} - 1 = \frac{y-2}{4}$

Rational Expression Operations

V. $\frac{5}{y+1} + \frac{2y}{y^2-1} = \frac{1}{y+1}$

Rational Expression Operations

Name: **Key** Date: _____

Question 1

Simplify

I. $\frac{8x^3}{5x} \cdot \frac{3}{4x}$

$\frac{8x^3}{5x} \cdot \frac{3}{4x} \longrightarrow \frac{(8x^3)(3)}{(5x)(4x)} \longrightarrow \frac{24x^3}{20x^2} \longrightarrow \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x}{2 \cdot 2 \cdot 5 \cdot x \cdot x} \longrightarrow \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x}{2 \cdot 2 \cdot 5 \cdot x \cdot x}$

multiply; prime factorization; cancel

$\downarrow$

$\frac{6x}{5}$

II. $\frac{3y^2z}{2} \cdot \frac{4}{yz^3}$

$\frac{3y^2z}{2} \cdot \frac{4}{yz^3} \longrightarrow \frac{(3y^2z)(4)}{(2)(yz^3)} \longrightarrow \frac{12y^2z}{2yz^3} \longrightarrow \frac{2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot z}{2 \cdot y \cdot z \cdot z \cdot z} \longrightarrow \frac{2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot z}{2 \cdot y \cdot z \cdot z \cdot z}$

multiply; prime factorization; cancel

$\downarrow$

$\frac{6y}{z^2}$

III. $\frac{x^2 - y^2}{4x + 4y} \cdot \frac{x + y}{x - y}$

$\frac{x^2 - y^2}{4x + 4y} \cdot \frac{x + y}{x - y} \longrightarrow \frac{(x^2 - y^2)(x + y)}{(4x + 4y)(x - y)} \longrightarrow \frac{(x - y)(x + y)(x + y)}{4(x + y)(x - y)} \longrightarrow \frac{(x - y)(x + y)(x + y)}{4(x + y)(x - y)}$

factor; cancel

$\downarrow$

$\frac{x + y}{4}$

Rational Expression Operations

IV. $\frac{x^2-6x+5}{x+6} \cdot \frac{x-6}{x^2+36}$

$\frac{x^2-6x+5}{x+6} \cdot \frac{x-6}{x^2+36} \longrightarrow \frac{(x^2-6x+5)(x-6)}{(x+6)(x^2+36)} \longrightarrow \frac{(x-5)(x-1)(x-6)}{(x+6)(x^2+36)}$

factor

V. $\frac{x^2+2x-3}{x^2+4x-5} \cdot \frac{x^2+3x-10}{x^2+5x+6}$

$\frac{x^2+2x-3}{x^2+4x-5} \cdot \frac{x^2+3x-10}{x^2+5x+6} \longrightarrow \frac{(x^2+2x-3)(x^2+3x-10)}{(x^2+4x-5)(x^2+5x+6)} \longrightarrow \frac{(x-1)(x+3)(x-2)(x+5)}{(x-1)(x+5)(x+2)(x+3)}$

factor

↓

$\frac{x-2}{x+2} \longleftarrow \frac{(x-1)(x+3)(x-2)(x+5)}{(x-1)(x+5)(x+2)(x+3)}$

cancel

VI. $\frac{y^2+4y+4}{(y-1)^2} \cdot \frac{y^2-2y+1}{(y+2)^2}$

$\frac{y^2+4y+4}{(y-1)^2} \cdot \frac{y^2-2y+1}{(y+2)^2} \longrightarrow \frac{(y^2+4y+4)(y^2-2y+1)}{(y-1)(y-1)(y+2)(y+2)} \longrightarrow \frac{(y+2)(y+2)(y-1)(y-1)}{(y-1)(y-1)(y+2)(y+2)}$

factor

↓

1

Rational Expression Operations

VII. $\frac{y}{4} \div \frac{5}{y}$

$\frac{y}{4} \div \frac{5}{y} \longrightarrow \frac{y}{4} \cdot \frac{y}{5} \longrightarrow \frac{(y)(y)}{(4)(5)} \longrightarrow \frac{y^2}{20}$

flip — multiply

VIII. $\frac{4x-8}{x+2} \div \frac{x-2}{x^2-4}$

$\frac{4x-8}{x+2} \div \frac{x-2}{x^2-4} \longrightarrow \frac{4(x-2)}{x+2} \div \frac{x-2}{(x-2)(x+2)} \longrightarrow \frac{4(x-2)}{x+2} \cdot \frac{(x-2)(x+2)}{x-2}$

factor — flip

$\downarrow$

$\frac{4(x-2)(x-2)(x+2)}{(x+2)(x-2)} \longleftarrow \frac{4(x-2)(x-2)(x+2)}{(x+2)(x-2)} \longleftarrow 4(x-2) \longleftarrow 4x-8$

cancel — distribute

IX. $\frac{x^2-14x+49}{2x^2-3x-14} \div \frac{3x^2-20x-7}{x^2-6x-16}$

$\frac{x^2-14x+49}{2x^2-3x-14} \div \frac{3x^2-20x-7}{x^2-6x-16} \longrightarrow \frac{(x-7)^2}{(x+2)(2x-7)} \div \frac{(x-7)(3x+1)}{(x+2)(x-8)}$

factor

$\downarrow$

$\frac{(x-7)(x-7)(x+2)(x-8)}{(x+2)(2x-7)(x-7)(3x+1)} \longleftarrow \frac{(x-7)^2}{(x+2)(2x-7)} \cdot \frac{(x+2)(x-8)}{(x-7)(3x+1)}$

flip

$\downarrow$

$\frac{(x-7)(x-7)(x+2)(x-8)}{(x+2)(2x-7)(x-7)(3x+1)} \longrightarrow \frac{(x-7)(x-8)}{(2x-7)(3x+1)}$

cancel

Rational Expression Operations

X. $\frac{3}{x} + \frac{5}{x}$

$\frac{3}{x} + \frac{5}{x} \longrightarrow \frac{8}{x}$

add

XI. $\frac{y}{12} + \frac{2y+5}{12}$

$\frac{y}{12} + \frac{2y+5}{12} \longrightarrow \frac{3y+5}{12}$

add

XII. $\frac{x^2-5x}{x^2+6x+9} + \frac{4x-12}{x^2+6x+9}$

$\frac{x^2-5x}{x^2+6x+9} + \frac{4x-12}{x^2+6x+9} \longrightarrow \frac{x^2-5x+4x-12}{x^2+6x+9} \longrightarrow \frac{x^2-x-12}{x^2+6x+9}$

add

$\downarrow$

$\frac{x-4}{x+3} \longleftarrow \frac{(x-4)(x+3)}{(x+3)(x+3)} \longleftarrow \frac{(x-4)(x+3)}{(x+3)(x+3)}$

cancel factor

XIII. $\frac{9}{2y+3} - \frac{5}{2y+3}$

$\frac{9}{2y+3} - \frac{5}{2y+3} \longrightarrow \frac{4}{2y+3}$

subtract

Rational Expression Operations

XIV. $\frac{2x^2+3x}{x^2-7x+12} - \frac{x^2+4x+6}{x^2-7x+12}$

$\frac{2x^2+3x}{x^2-7x+12} - \frac{x^2+4x+6}{x^2-7x+12} \longrightarrow \frac{(2x^2+3x)-(x^2+4x+6)}{x^2-7x+12} \longrightarrow \frac{(2x^2+3x)+(-x^2-4x-6)}{x^2-7x+12}$

convert to opposite and add

$\frac{x+2}{x-4} \longleftarrow \frac{(x-3)(x+2)}{(x-4)(x-3)} \longleftarrow \frac{(x-3)(x+2)}{(x-4)(x-3)} \longleftarrow \frac{1x^2-1x-6}{x^2-7x+12}$

cancel — factor — add

XV. $\frac{3}{x^2} + \frac{4}{x}$

$\frac{3}{x^2} + \frac{4}{x} \longrightarrow \frac{3}{x^2} + \frac{4}{x} \cdot \frac{x}{x} \longrightarrow \frac{3}{x^2} + \frac{4x}{x^2} \longrightarrow \frac{3+4x}{x^2}$

LCD — multiply

XVI. $\frac{3}{x+2} + \frac{7}{(x+2)^2}$

$\frac{3}{x+2} + \frac{7}{(x+2)^2} \longrightarrow \frac{x+2}{x+2} \cdot \frac{3}{x+2} + \frac{7}{(x+2)^2} \longrightarrow \frac{3x+6}{(x+2)^2} + \frac{7}{(x+2)^2} \longrightarrow \frac{3x+13}{(x+2)^2}$

LCD — multiply — add

XVII. $\frac{x}{x-1} - \frac{x-1}{x}$

$\frac{x}{x-1} - \frac{x-1}{x} \longrightarrow \frac{x}{x} \cdot \frac{x}{x-1} - \frac{x-1}{x} \cdot \frac{x-1}{x-1} \longrightarrow \frac{x^2}{x(x-1)} - \frac{(x-1)^2}{x(x-1)} \longrightarrow \frac{x^2}{x(x-1)} - \frac{x^2-2x+1}{x(x-1)}$

LCD — multiply — factor

$\frac{2x-1}{x(x-1)} \longleftarrow \frac{(x^2)+(-x^2+2x-1)}{x(x-1)} \longleftarrow \frac{(x^2)-(x^2-2x+1)}{x(x-1)}$

convert to opposite and add

Rational Expression Operations

XVIII. $\frac{x}{x^2+2x+1} + \frac{1}{x^2+5x+4}$

$\frac{x}{x^2+2x+1} + \frac{1}{x^2+5x+4} \longrightarrow \frac{x}{(x+1)^2} + \frac{1}{(x+1)(x+4)}$

factor

$\frac{x+4}{x+4} \cdot \frac{x}{(x+1)^2} + \frac{1}{(x+1)(x+4)} \cdot \frac{x+1}{x+1}$

LCD

$\frac{x^2+4x}{(x+4)(x+1)^2} + \frac{x+1}{(x+4)(x+1)^2}$

multiply

$\frac{x^2+5x+1}{(x+4)(x+1)^2}$

add

XIX. $\frac{7}{x^2+25x+24} - \frac{0}{x^2+15x+56}$

$\frac{7}{x^2+25x+24} - \frac{0}{x^2+15x+56} \longrightarrow \frac{7}{x^2+25x+24} - 0 \longrightarrow \frac{7}{x^2+25x+24}$

$\frac{7}{(x+1)(x+24)}$

factor

Rational Expression Operations

XX. $2 + \frac{1}{5-x}$

$2 + \frac{1}{5-x} \longrightarrow \frac{2}{1} + \frac{1}{5-x} \longrightarrow \frac{5-x}{5-x} \cdot \frac{2}{1} + \frac{1}{5-x} \longrightarrow \frac{10-2x}{5-x} + \frac{1}{5-x}$

LCD

multiply

$\downarrow$

$-\frac{2x-11}{5-x} \longleftarrow \frac{-2x+11}{5-x} \longleftarrow \frac{11-2x}{5-x} \longleftarrow \frac{10-2x+1}{5-x}$

factor

descending order

add

XXI. $\frac{5y}{4} + \frac{y-2}{-4}$

$\frac{5y}{4} + \frac{y-2}{-4} \longrightarrow \frac{5y}{4} + \frac{y-2}{-4} \cdot \frac{-1}{-1} \longrightarrow \frac{5y}{4} + \frac{-y+2}{4} \longrightarrow \frac{5y-y+2}{4} \longrightarrow \frac{4y+2}{4}$

change to opposite

add

$\downarrow$

$\frac{2y+1}{2}$

simplify

XXII. $\frac{x-5}{x^2-64} - \frac{5-x}{64-x^2}$

$\frac{x-5}{x^2-64} - \frac{5-x}{64-x^2} \longrightarrow \frac{x-5}{x^2-64} - \frac{5-x}{64-x^2} \cdot \frac{-1}{-1} \longrightarrow \frac{x-5}{x^2-64} - \frac{-5+x}{x^2-64}$

change to opposite

$\downarrow$

$0 \longleftarrow \frac{(x-5)+(5-x)}{x^2-64} \longleftarrow \frac{(x-5)-(-5+x)}{x^2-64}$

add

convert to opposite and add

Rational Expression Operations

XXIII. $\frac{3y+2}{3y+6} + \frac{y}{4-y^2}$

$\frac{3y+2}{3y+6} + \frac{y}{4-y^2} \longrightarrow \frac{3y+2}{3(y+2)} + \frac{y}{(2+y)(2-y)} \longrightarrow \frac{3y+2}{3(y+2)} + \frac{y}{(y+2)(2-y)}$

factor

associative/commutative

$\downarrow$

$\frac{(2-y)(3y+2)}{3(y+2)(2-y)} + \frac{3y}{3(y+2)(2-y)} \longleftarrow \frac{2-y}{2-y} \cdot \frac{3y+2}{3(y+2)} + \frac{y}{(y+2)(2-y)} \cdot \frac{3}{3}$

multiply

LCD

$\downarrow$

$\frac{-3y^2+4y+4}{3(y+2)(2-y)} + \frac{3y}{3(y+2)(2-y)} \longrightarrow \frac{-3y^2+4y+4+3y}{3(y+2)(2-y)}$

FOIL

$\downarrow$

$-\frac{3y^2-7y-4}{3(y+2)(2-y)} \longleftarrow \frac{-3y^2+7y+4}{3(y+2)(2-y)}$

factor

add

XXIV. $\dfrac{\frac{2}{x^2y} + \frac{3}{xy^2}}{\frac{3}{xy^2} + \frac{2}{x^2y}}$

$\dfrac{\frac{2}{x^2y} + \frac{3}{xy^2}}{\frac{3}{xy^2} + \frac{2}{x^2y}} \longrightarrow \dfrac{\frac{2}{x^2y} + \frac{3}{xy^2}}{\frac{2}{x^2y} + \frac{3}{xy^2}} \longrightarrow 1$

associative/commutative

Rational Expression Operations

XXV.

$$\frac{\frac{z}{z^2+3z-4}-\frac{1}{z^2+3z-4}}{\frac{z}{z^2+6z+8}+\frac{3}{z^2+6z+8}}$$

$$\frac{\frac{z}{z^2+3z-4}-\frac{1}{z^2+3z-4}}{\frac{z}{z^2+6z+8}+\frac{3}{z^2+6z+8}} \longrightarrow \frac{\frac{z}{(z-1)(z+4)}-\frac{1}{(z-1)(z+4)}}{\frac{z}{(z+2)(z+4)}+\frac{3}{(z+2)(z+4)}}$$ factor

$$\frac{\frac{z}{(z-1)(z+4)}-\frac{1}{(z-1)(z+4)}}{\frac{z}{(z+2)(z+4)}+\frac{3}{(z+2)(z+4)}} \longleftarrow \frac{\frac{z-1}{(z-1)(z+4)}}{\frac{z+3}{(z+2)(z+4)}}$$ add and subtract

$$\frac{z-1}{(z-1)(z+4)} \cdot \frac{(z+2)(z+4)}{z+3}$$

multiply the reciprocal

$$\frac{(z-1)(z+2)(z+4)}{(z-1)(z+4)(z+3)} \longrightarrow \frac{\cancel{(z-1)}(z+2)\cancel{(z+4)}}{\cancel{(z-1)}\cancel{(z+4)}(z+3)} \longrightarrow \frac{z+2}{z+3}$$

multiply — cancel

XXVI.

$$\frac{y-3-\frac{2}{y}}{y-4-\frac{3}{y}}$$

$$\frac{y-3-\frac{2}{y}}{y-4-\frac{3}{y}} \longrightarrow \frac{\frac{y^2-3y-2}{y}}{\frac{y^2-4y-3}{y}} \longrightarrow \frac{y^2-3y-2}{y} \cdot \frac{y}{y^2-4y-3} \longrightarrow \frac{(y^2-3y-2)(y)}{(y)(y^2-4y-3)}$$

LCD — multiply the reciprocal — multiply

$$\frac{y^2-3y-2}{y^2-4y-3} \longleftarrow \frac{(y^2-3y-2)\cancel{(y)}}{\cancel{(y)}(y^2-4y-3)}$$

cancel

Rational Expression Operations

XXVII. $\dfrac{x + x^{-1}}{x - x^{-1}} \longrightarrow \dfrac{x + \frac{1}{x}}{x - \frac{1}{x}} \longrightarrow \dfrac{\frac{x^2}{x} + \frac{1}{x}}{\frac{x^2}{x} + \frac{1}{x}} \longrightarrow \dfrac{\frac{x^2+1}{x}}{\frac{x^2-1}{x}} \longrightarrow \dfrac{x^2+1}{x} \cdot \dfrac{x}{x^2-1}$

convert LCD add multiply the reciprocal

$\downarrow$

$\dfrac{(x^2+1)(x)}{(x)(x^2-1)} \longleftarrow \dfrac{(x^2+1)(x)}{(x)(x^2-1)} \longleftarrow \dfrac{x^2+1}{x^2-1} \longleftarrow \dfrac{x^2+1}{(x+1)(x-1)}$

multiply cancel factor

Rational Expression Operations

Question 2

Solve

I. $\frac{5}{8} - \frac{3}{5} = \frac{x}{10}$

$\frac{5}{8} - \frac{3}{5} = \frac{x}{10} \longrightarrow 10\left(\frac{5}{8} - \frac{3}{5}\right) = x \longrightarrow \frac{1}{4} = x$

multiply — multiply

II. $\frac{y}{6} - \frac{6}{y} = 0$

$\frac{y}{6} - \frac{6}{y} = 0 \longrightarrow (6y)\frac{y}{6} - \frac{6}{y}(6y) \longrightarrow y^2 - 36 = 0 \longrightarrow (y+6)(y-6) = 0$

multiply LCD — factor

$y + 6 = 0$ ⟵ $y - 6 = 0$

true $y = -6$

true $y = 6$

check

III. $\frac{x-4}{x+6} = \frac{1}{3}$

$\frac{x-4}{x+6} = \frac{1}{3} \longrightarrow (3)(x-4) = (1)(x+6) \longrightarrow 3x - 12 = x + 6 \longrightarrow 2x - 12 = 6 \longrightarrow 2x = 18$

cross multiply — combine like terms

$x = 9$

IV. $\frac{y+2}{5} - 1 = \frac{y-2}{4}$

$\frac{y+2}{5} - 1 = \frac{y-2}{4} \longrightarrow \frac{y+2}{5} - \frac{5}{5} = \frac{y-2}{4} \longrightarrow \frac{y-3}{5} = \frac{y-2}{4} \longrightarrow (5)(y-2) = (y-3)(4)$

LCD — subtract — cross multiply

$y = -2 \longleftarrow y - 10 = -12 \longleftarrow 5y - 10 = 4y - 12$

combine like terms

Rational Expression Operations

V. $\frac{5}{y+1} + \frac{2y}{y^2-1} = \frac{1}{y+1}$

$\frac{5}{y+1} + \frac{2y}{y^2-1} = \frac{1}{y+1} \longrightarrow \frac{5}{y+1} + \frac{2y}{(y+1)(y-1)} = \frac{1}{y+1} \longrightarrow \frac{5y-5}{(y+1)(y-1)} + \frac{2y}{(y+1)(y-1)} = \frac{1}{y+1}$

factor

LCD

$(y+1)(y-1) \cdot \frac{7y-5}{(y+1)(y-1)} = \frac{1}{y+1} \cdot (y+1)(y-1) \longleftarrow \frac{7y-5}{(y+1)(y-1)} = \frac{1}{y+1}$

multiply LCD

add

$(y+1)(y-1) \cdot \frac{7y-5}{(y+1)(y-1)} = \frac{1}{y+1} \cdot (y+1)(y-1) \longrightarrow 7y-5 = y-1$

cancel

$y = 2/3 \longleftarrow y = 4/6 \longleftarrow 6y = 4 \longleftarrow 6y-5 = -1$

simplify

combine like terms