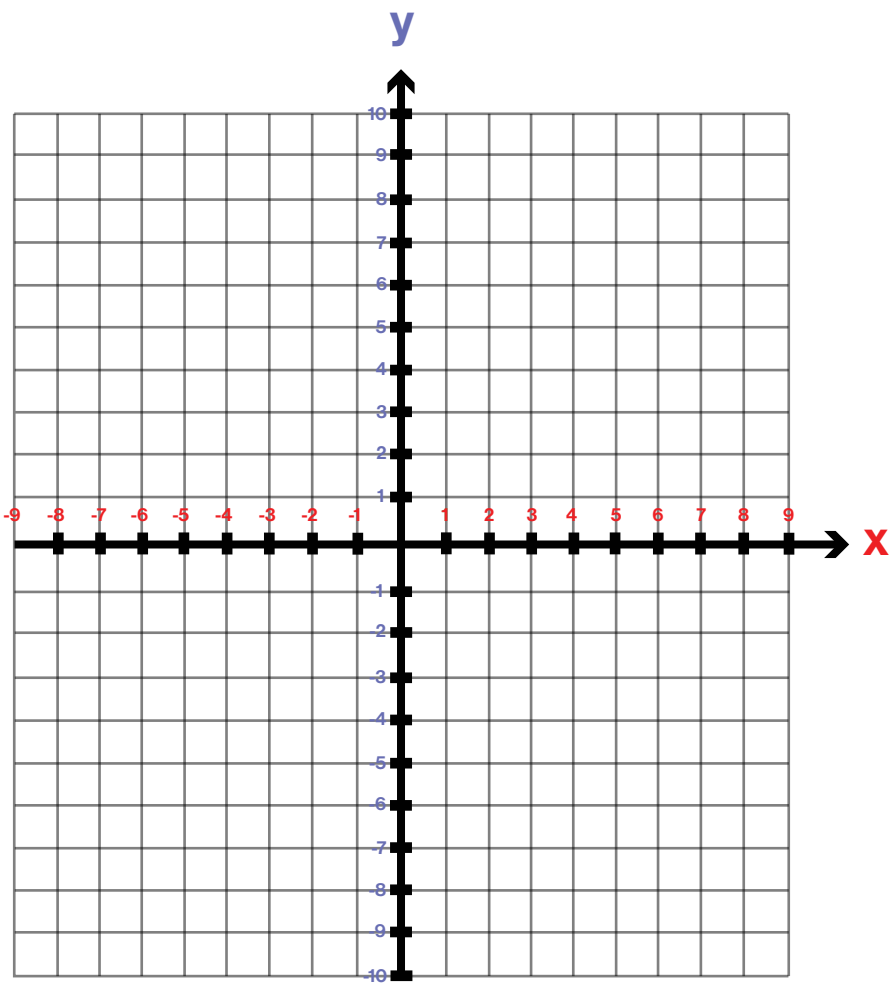
# Limits (Infinity)

Name: _____ Date: _____

## Question 1

Solve

I. $\lim_{x \to \infty} \frac{3-(2/x)}{4+(\sqrt{2}/x^2)} =$



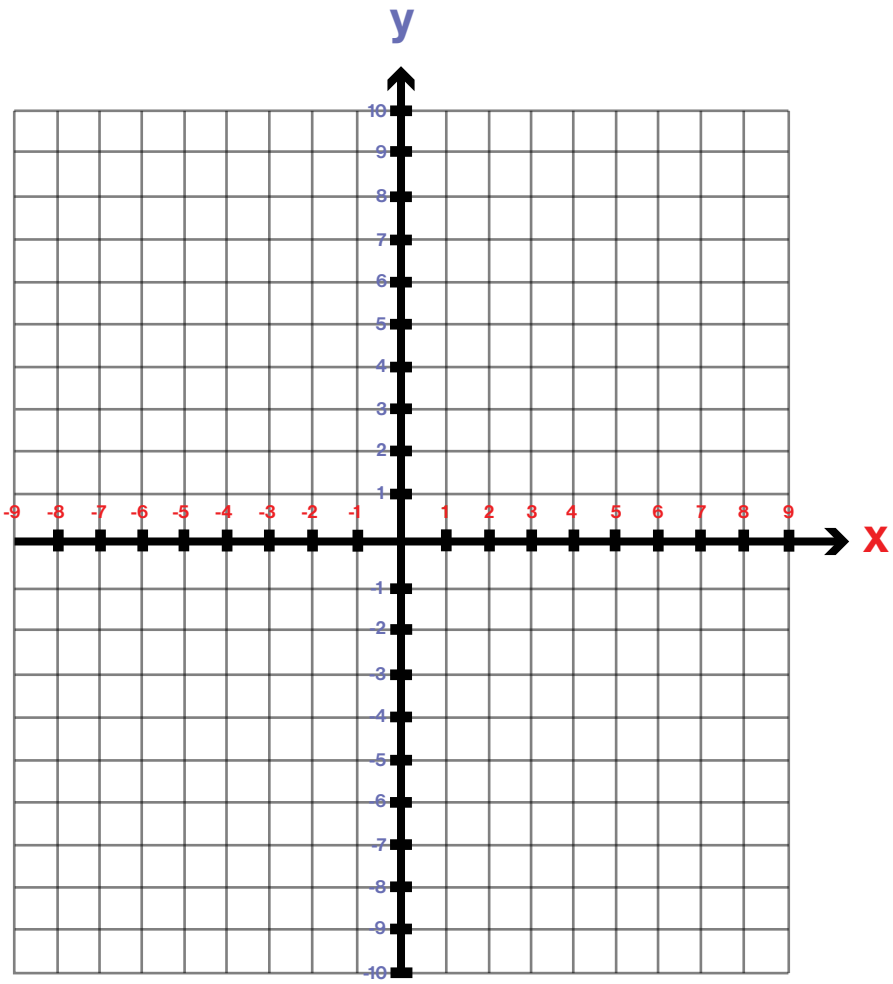

II. $\lim_{x \to -\infty} \frac{3-(2/x)}{4+(\sqrt{2}/x^2)} =$

# Limits (Infinity)

## Question 2

Solve

I. $\lim_{x \to -\infty} \frac{\cos x}{3x} =$



# Limits (Infinity)

## Question 3

Solve (Check by Graphing)

I. $\lim_{x \to \infty} \frac{7x^3}{x^3 - 3x^2 + 6x} =$

II. $\lim_{x \to -\infty} \frac{7x^3}{x^3 - 3x^2 + 6x} =$

III. $\lim_{x \to \infty} \frac{5x^8 - 2x^3 + 9}{3 + x - 4x^5} =$

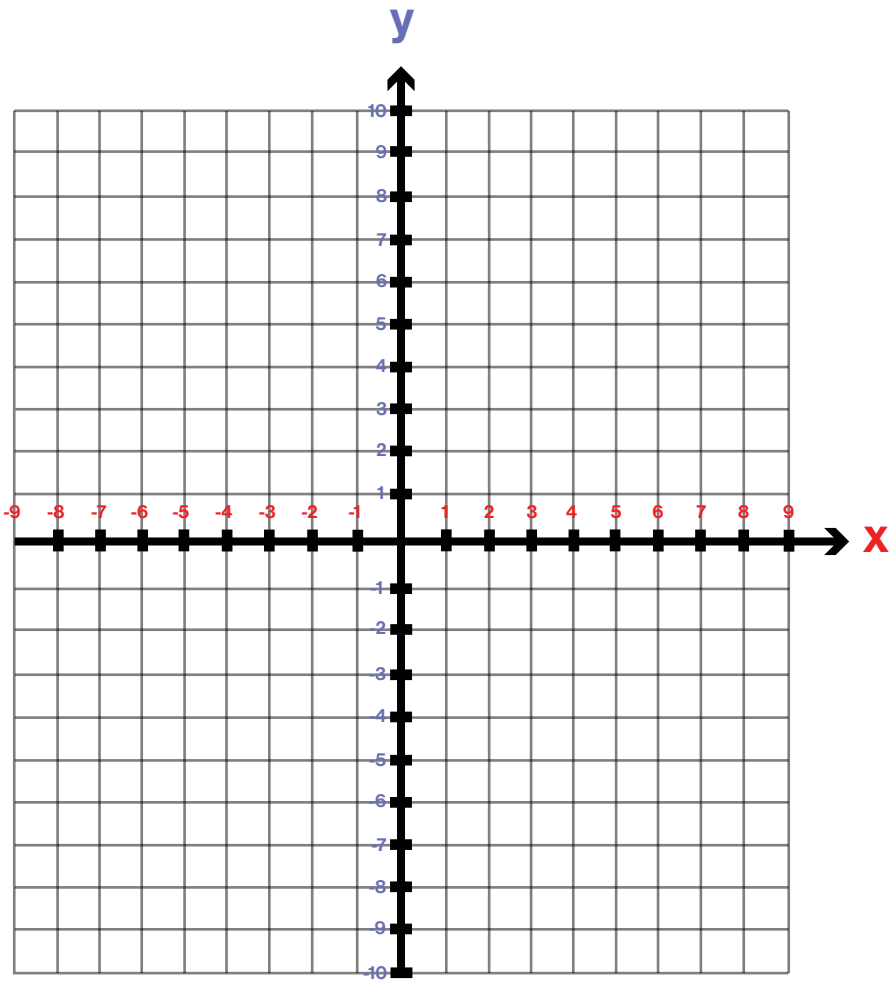
IV. $\lim_{x \to -\infty} \frac{5x^8 - 2x^3 + 9}{3 + x - 4x^5} =$

# Limits (Infinity)

## Question 4

Solve

I. $\lim_{x \to 0^+} \frac{1}{3x} =$

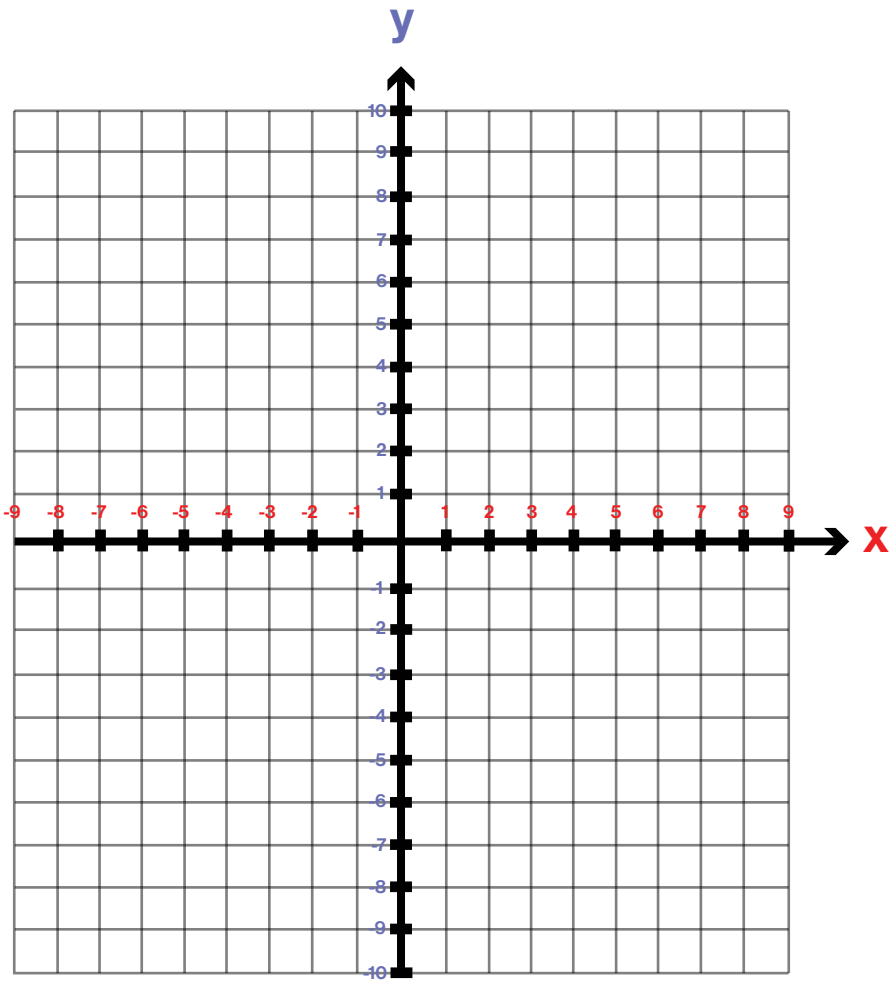


# Limits (Infinity)

## Question 5

Solve

I. $\lim_{x \to 0^-} (1 + \csc x) =$



# Limits (Infinity)

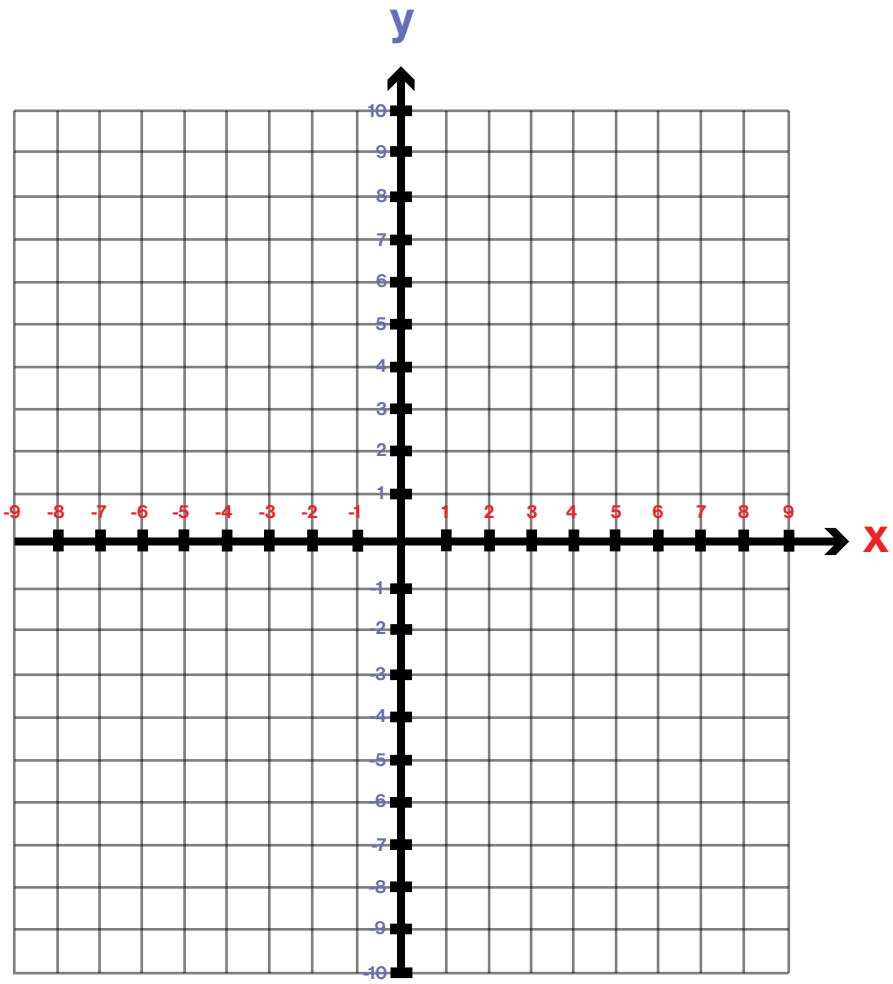
## Question 6

Make a graph to meet the conditions

I. $f(0) = 0$ $f(1) = 2$ $f(-1) = -2$

$\lim_{x \to -\infty} f(x) = -1$

$\lim_{x \to \infty} f(x) = 1$



# Limits (Infinity)

## Question 7

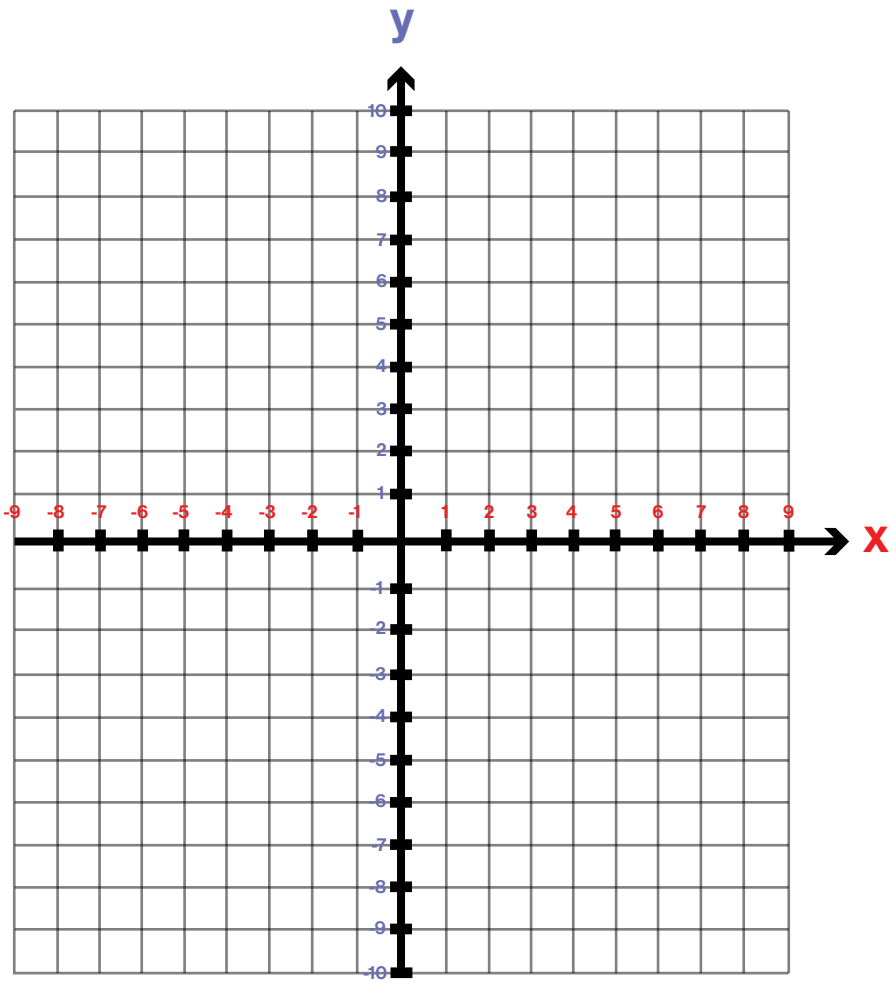
Make a graph to meet the conditions

I.

$\lim_{x \to \pm\infty} f(x) = 1$

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$



# Limits (Infinity)

## Question 8

Solve (Check by Graphing)

I. $\lim_{x \to \infty} \left( \sqrt{x^2 + 25} - \sqrt{x^2 - 1} \right)$

II. $\lim_{x \to -\infty} \left( \sqrt{x^2 + 3} + x \right)$

# Limits (Infinity)

III. $\lim_{x \to -\infty} \left( 2x + \neg\sqrt{4x^2 + 3x - 2} \right)$

IV. $\lim_{x \to \infty} \left( \neg\sqrt{9x^2 - x} - 3x \right)$

# Limits (Infinity)

V. $\lim_{x \to \infty} \left( \sqrt{x^2 + 3x} - \sqrt{x^2 - 2x} \right)$

VI. $\lim_{x \to \infty} \left( \sqrt{x^2 + x} - \sqrt{x^2 - x} \right)$

# Limits (Infinity)

Name: **Key** Date: _____

## Question 1

Solve

I. $\lim_{x \to \infty} \frac{3-(2/x)}{4+(\sqrt{2}/x^2)} = \frac{3}{4}$

$$\frac{3-(2/(1{,}000{,}000))}{4+(\sqrt{2}/(1{,}000{,}000)^2)} \approx 0.75$$

$$\downarrow$$

$$\frac{3-0}{4+0} = \frac{3}{4}$$

$$\frac{3}{4} = 0.75$$

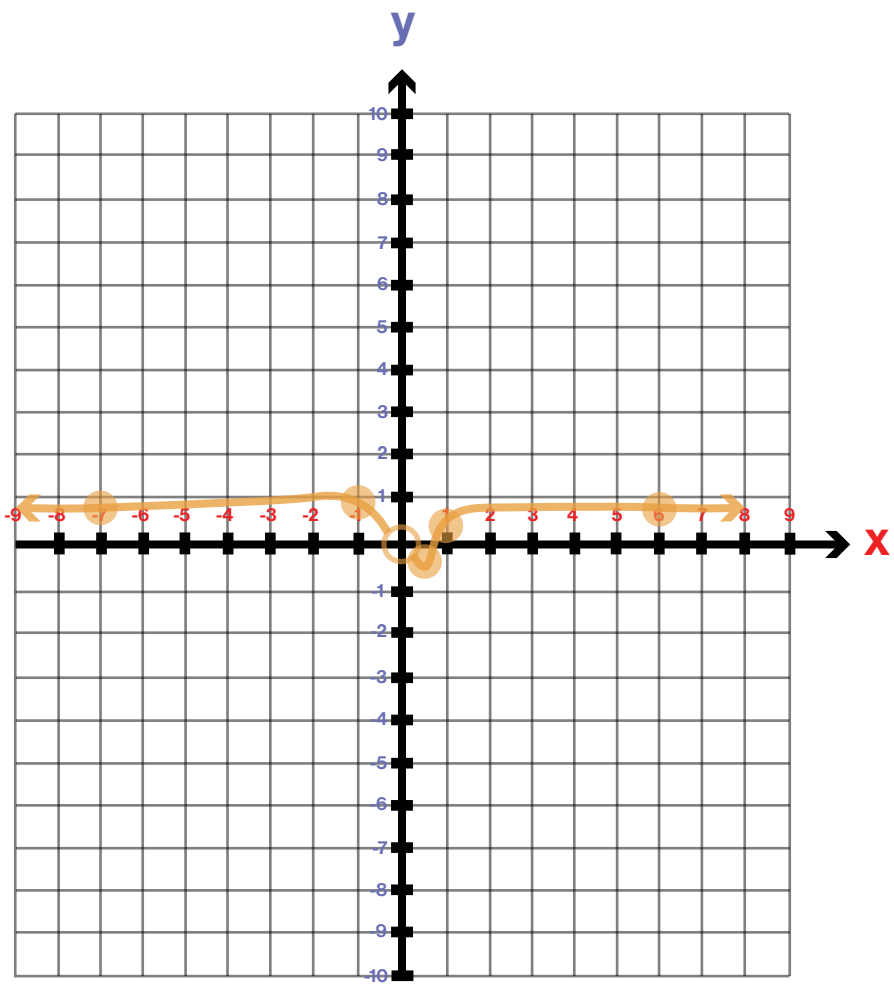
II. $\lim_{x \to -\infty} \frac{3-(2/x)}{4+(\sqrt{2}/x^2)} = \frac{3}{4}$

$$\frac{3-(2/(-1{,}000{,}000))}{4+(\sqrt{2}/(-1{,}000{,}000)^2)} \approx 0.75$$

$$\downarrow$$

$$\frac{3-0}{4+0} = \frac{3}{4}$$

$$\frac{3}{4} = 0.75$$



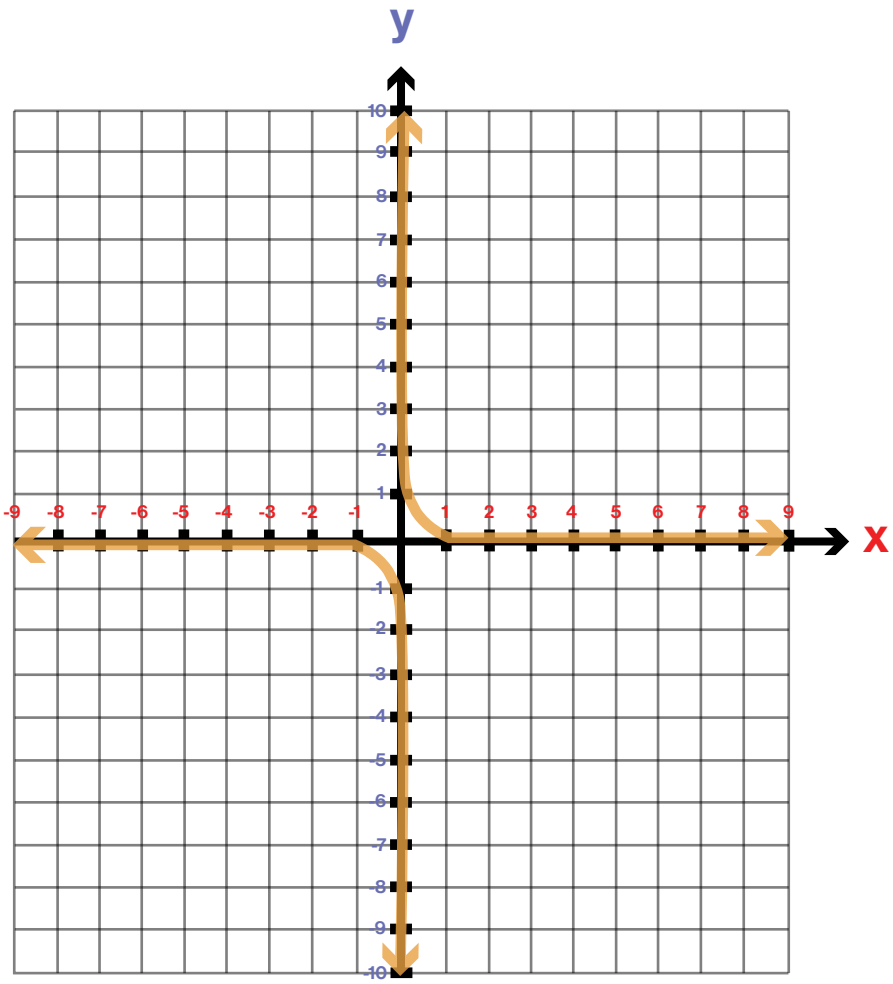
# Limits (Infinity)

## Question 2

Solve

I. $\lim_{x \to -\infty} \frac{\cos x}{3x} = 0$

$$\frac{\cos(-1{,}000{,}000)}{3(-1{,}000{,}000)} \approx 0$$



# Limits (Infinity)

## Question 3

Solve (Check by Graphing)

I. $\lim_{x \to \infty} \frac{7x^3}{x^3 - 3x^2 + 6x} = 7$

$$\frac{7(1{,}000)^3}{(1{,}000)^3 - 3(1{,}000)^2 + 6(1{,}000)} \approx 7$$

$$\frac{7(1{,}000{,}000)^3}{(1{,}000{,}000)^3 - 3(1{,}000{,}000)^2 + 6(1{,}000{,}000)} \approx 7$$

II. $\lim_{x \to -\infty} \frac{7x^3}{x^3 - 3x^2 + 6x} = 7$

$$\frac{7(-1{,}000)^3}{(-1{,}000)^3 - 3(-1{,}000)^2 + 6(-1{,}000)} \approx 7$$

$$\frac{7(-1{,}000{,}000)^3}{(-1{,}000{,}000)^3 - 3(-1{,}000{,}000)^2 + 6(-1{,}000{,}000)} \approx 7$$

III. $\lim_{x \to \infty} \frac{5x^8 - 2x^3 + 9}{3 + x - 4x^5} = -\infty$

$$\frac{5(10)^8 - 2(10)^3 + 9}{3 + (10) - 4(10)^5} \approx -1{,}250$$

$$\frac{5(100)^8 - 2(100)^3 + 9}{3 + (100) - 4(100)^5} \approx -1{,}250{,}000$$

$$\frac{5(1{,}000)^8 - 2(1{,}000)^3 + 9}{3 + (1{,}000) - 4(1{,}000)^5} \approx -1{,}250{,}000{,}000$$

IV. $\lim_{x \to -\infty} \frac{5x^8 - 2x^3 + 9}{3 + x - 4x^5} = \infty$

$$\frac{5(-10)^8 - 2(-10)^3 + 9}{3 + (-10) - 4(-10)^5} \approx 1{,}250$$

$$\frac{5(-100)^8 - 2(-100)^3 + 9}{3 + (-100) - 4(-100)^5} \approx 1{,}250{,}000$$

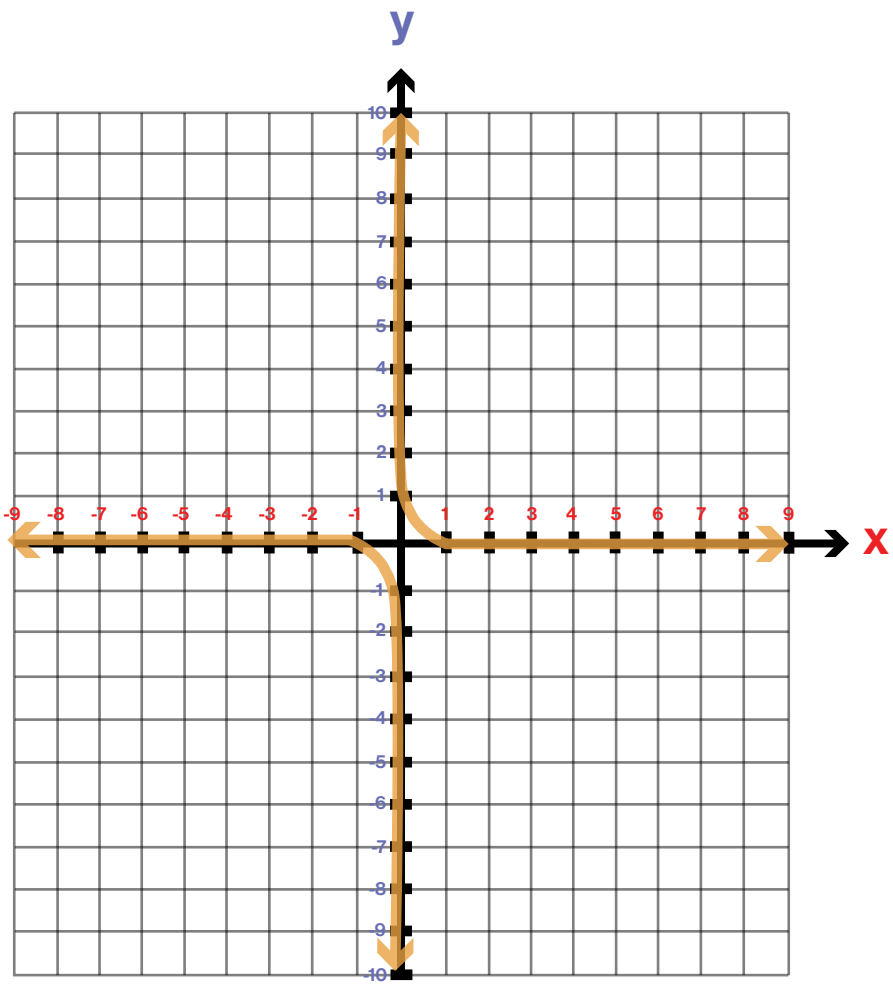
$$\frac{5(-1{,}000)^8 - 2(-1{,}000)^3 + 9}{3 + (-1{,}000) - 4(-1{,}000)^5} \approx 1{,}250{,}000{,}000$$

# Limits (Infinity)

## Question 4

Solve

I. $\lim_{x \to 0^+} \frac{1}{3x} = \infty$

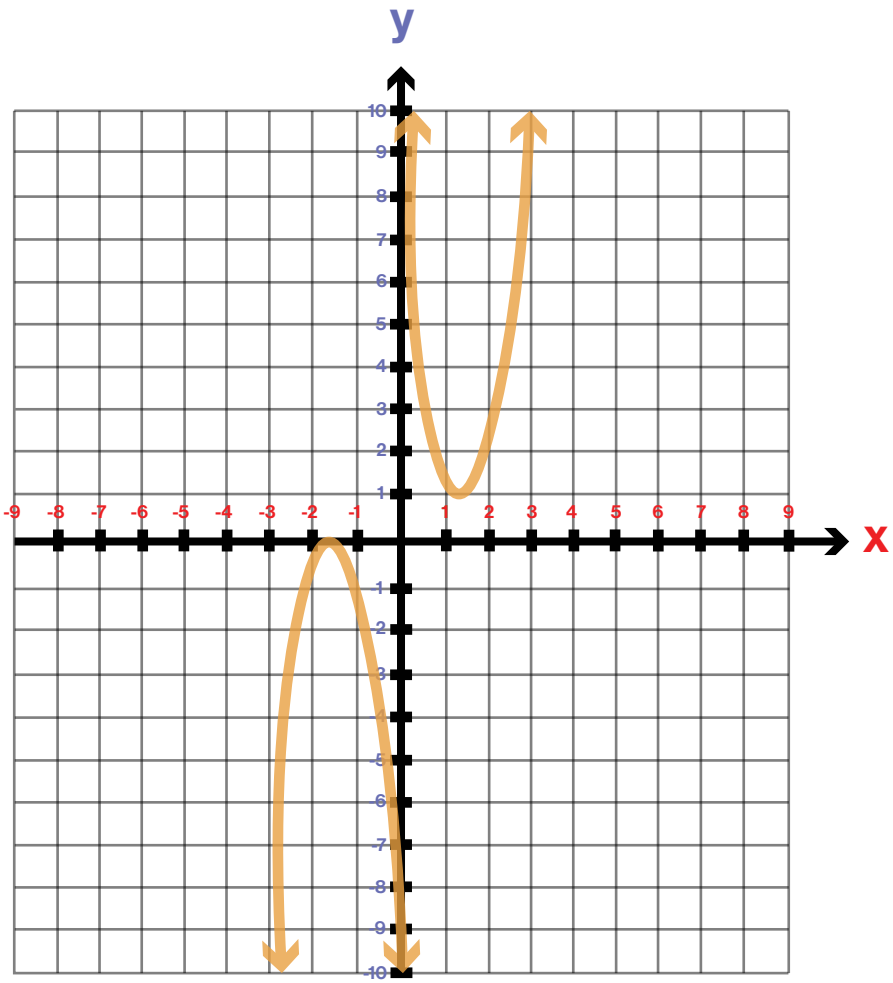


# Limits (Infinity)

## Question 5

Solve

I. $\lim_{x \to 0^-} (1 + \csc x) = -\infty$



# Limits (Infinity)

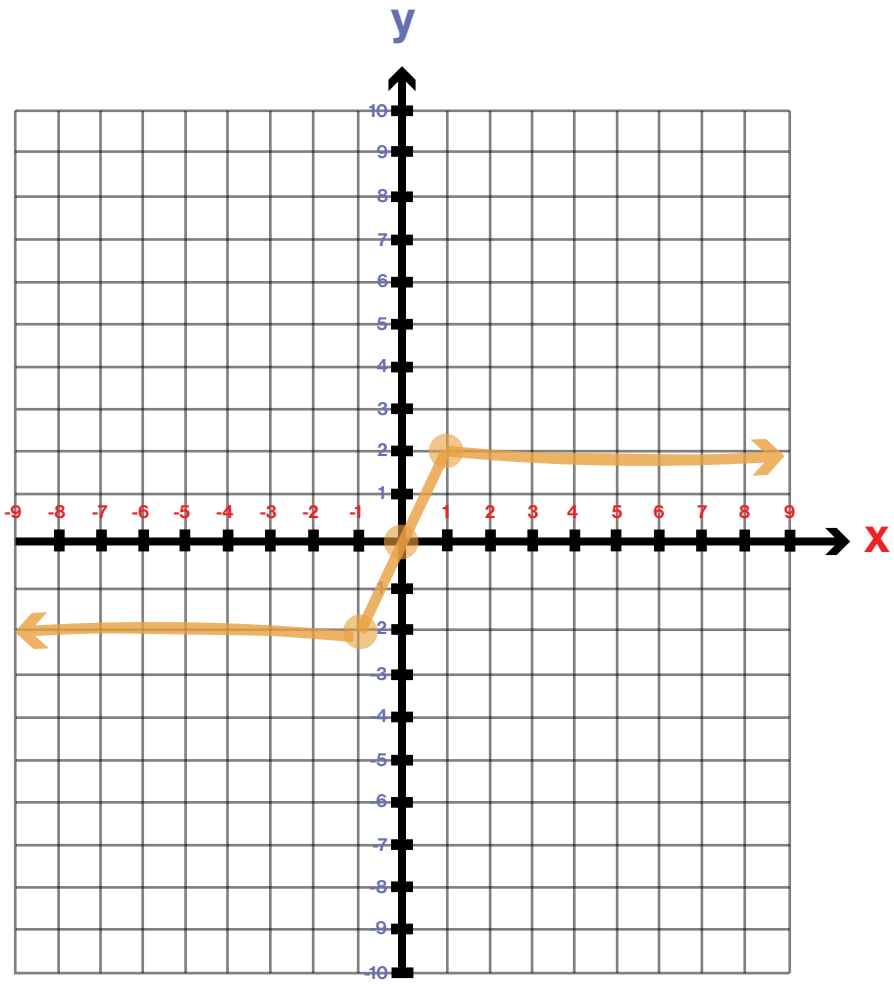
## Question 6

Make a graph to meet the conditions

$f(0) = 0$ $f(1) = 2$ $f(-1) = -2$

$\lim_{x \to -\infty} f(x) = -1$

$\lim_{x \to \infty} f(x) = 1$



# Limits (Infinity)

## Question 7

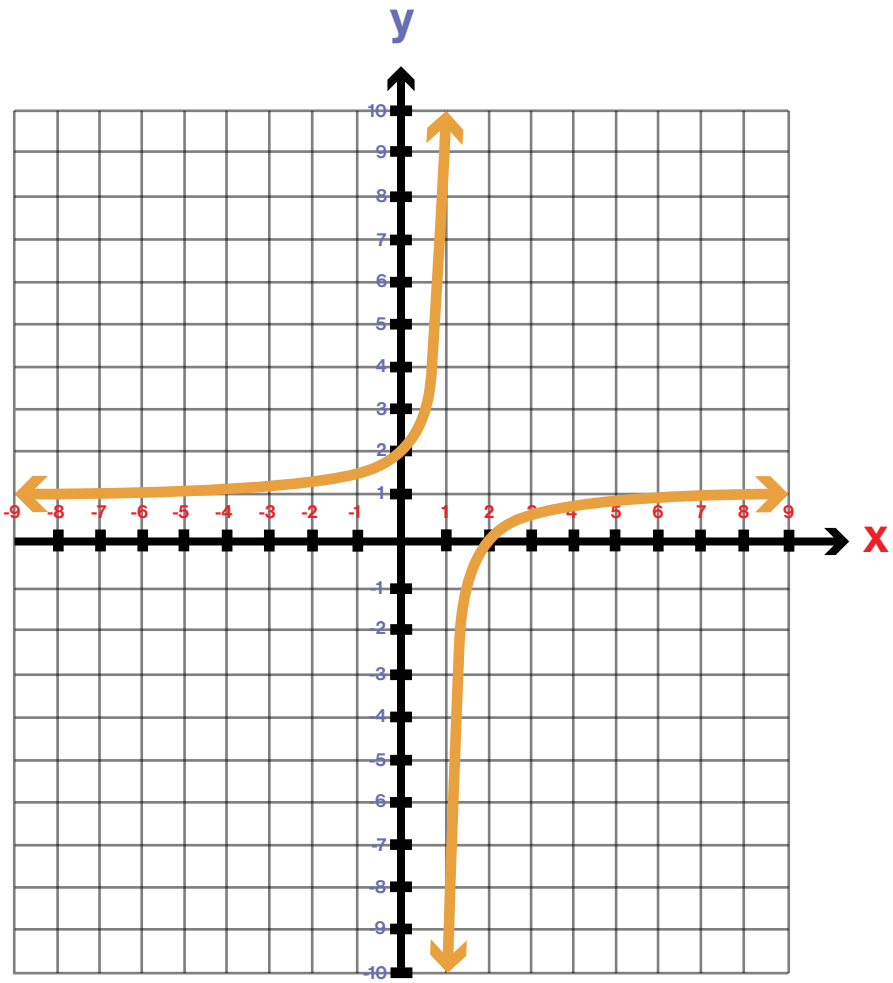
Make a graph to meet the conditions

I.

$\lim_{x \to \pm\infty} f(x) = 1$

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$




$$f(x) = 1 - \frac{1}{x - 1}$$

# Limits (Infinity)

## Question 8

Solve (Check by Graphing)

I. $\lim_{x \to \infty} \left(\sqrt{x^2+25} - \sqrt{x^2-1}\right) \longrightarrow \left(\sqrt{x^2+25} - \sqrt{x^2-1}\right)\left(\dfrac{\sqrt{x^2+25} + \sqrt{x^2-1}}{\sqrt{x^2+25} + \sqrt{x^2-1}}\right)$

$\downarrow$

$\dfrac{(x^2+25) - (x^2-1)}{\sqrt{x^2+25} + \sqrt{x^2-1}} \longrightarrow \dfrac{26}{\sqrt{x^2+25} + \sqrt{x^2-1}}$

$\downarrow$

$\dfrac{\frac{26}{x}}{\sqrt{1+\frac{25}{x^2}} + \sqrt{1-\frac{1}{x^2}}} \longrightarrow \dfrac{0}{1+1} \longrightarrow 0$

II. $\lim_{x \to -\infty} \left(\sqrt{x^2+3} + x\right) \longrightarrow \left(\sqrt{x^2+3} + x\right)\left(\dfrac{\sqrt{x^2+3} - x}{\sqrt{x^2+3} - x}\right) \longrightarrow \dfrac{(x^2+3) - (x^2)}{\sqrt{x^2+3} - x}$

$\downarrow$

$\dfrac{3}{\sqrt{x^2+3} - x} \longrightarrow \dfrac{\frac{3}{\sqrt{x^2}}}{\sqrt{1+\frac{3}{x^2}} - \frac{x}{\sqrt{x^2}}} \longrightarrow \dfrac{\frac{-3}{x}}{\sqrt{1+\frac{3}{x^2}} + 1}$

$\downarrow$

$\dfrac{0}{1+1} \longrightarrow 0$

# Limits (Infinity)

III. $\lim_{x \to -\infty} \left(2x + \sqrt{4x^2+3x-2}\right) \longrightarrow \left(2x + \sqrt{4x^2+3x-2}\right)\left(\frac{2x - \sqrt{4x^2+3x-2}}{2x - \sqrt{4x^2+3x-2}}\right) \longrightarrow \frac{(4x^2)-(4x^2+3x-2)}{2x - \sqrt{4x^2+3x-2}}$

$\downarrow$

$\frac{-3x+2}{2x - \sqrt{4x^2+3x-2}} \longleftarrow \frac{\frac{-3x+2}{\sqrt{x^2}}}{\frac{2x}{\sqrt{x^2}} - \sqrt{4+\frac{3}{x}-\frac{2}{x^2}}} \longleftarrow \frac{\frac{-3x+2}{-x}}{\frac{2x}{-x} - \sqrt{4+\frac{3}{x}-\frac{2}{x^2}}}$

$\downarrow$

$\frac{3-\frac{2}{x}}{-2-\sqrt{4+\frac{3}{x}-\frac{2}{x^2}}} \longrightarrow \frac{3-0}{-2-2} \longrightarrow \frac{-3}{4}$

IV. $\lim_{x \to \infty} \left(\sqrt{9x^2-x} - 3x\right) \longrightarrow \left(\sqrt{9x^2-x} - 3x\right)\left(\frac{\sqrt{9x^2-x} + 3x}{\sqrt{9x^2-x} + 3x}\right) \longrightarrow \frac{(9x^2-x) - (9x^2)}{\sqrt{9x^2-x} + 3x}$

$\downarrow$

$\frac{-x}{\sqrt{9x^2-x} + 3x} \longleftarrow \frac{\frac{-x}{x}}{\sqrt{\frac{9x^2}{x^2}-\frac{x}{x^2}} + \frac{3x}{x}} \longleftarrow \frac{-1}{\sqrt{9-\frac{1}{x}} + 3}$

$\downarrow$

$\frac{-1}{3+3} \longrightarrow \frac{-1}{6}$

# Limits (Infinity)

V. $\lim_{x \to \infty} \left(\sqrt{x^2+3x} - \sqrt{x^2-2x}\right) \longrightarrow \left(\sqrt{x^2+3x} - \sqrt{x^2-2x}\right)\left(\frac{\sqrt{x^2+3x} + \sqrt{x^2-2x}}{\sqrt{x^2+3x} + \sqrt{x^2-2x}}\right)$

$$\downarrow$$

$$\frac{(x^2+3x) - (x^2-2x)}{\sqrt{x^2+3x} + \sqrt{x^2-2x}} \longleftarrow \frac{5x}{\sqrt{x^2+3x} + \sqrt{x^2-2x}}$$

$$\downarrow$$

$$\frac{5}{\sqrt{1+\frac{3}{x}} + \sqrt{1-\frac{2}{x}}} \longrightarrow \frac{5}{1+1} \longrightarrow \frac{5}{2}$$

VI. $\lim_{x \to \infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right) \longrightarrow \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)\left(\frac{\sqrt{x^2+x} + \sqrt{x^2-x}}{\sqrt{x^2+x} + \sqrt{x^2-x}}\right)$

$$\downarrow$$

$$\frac{(x^2+x) - (x^2-x)}{\sqrt{x^2+x} + \sqrt{x^2-x}} \longleftarrow \frac{2x}{\sqrt{x^2+x} + \sqrt{x^2-x}}$$

$$\downarrow$$

$$\frac{2}{\sqrt{1+\frac{1}{x}} + \sqrt{1-\frac{1}{x}}} \longrightarrow \frac{2}{1+1} \longrightarrow 1$$