Exponent Rules

Natural Number Exponents

Treat natural number exponents as repeated multiplication.

$$\text{base} \longrightarrow a^{x} \longleftarrow \text{exponent}$$

Example:

$$7^{3} = 7 \cdot 7 \cdot 7 = 343$$

Fractional Exponents

(1) A fractional exponent can be treated as a radical (Fractional Exponent Rule).
(2) The base can be turned into its exponential form and can cancel out the fractional exponent (Power Rule).
***Note: If zero is the denominator in the fractional exponent, the product is undefined.**

$$a^{\frac{x}{y}}$$

Example 1:

$$64^{\frac{1}{2}} = \sqrt[2]{64^{1}} = 8$$

Example 2:

$$64^{\frac{1}{2}} = (8^{2})^{\frac{1}{2}} = (8^{\cancel{2}})^{\frac{1}{\cancel{2}}} = 8$$

Exponent Rules

Negative Exponents

Negative exponents should be converted into fraction form before solving (Negative Exponent Rule).

$$a^{-x} = \frac{1}{a^{x}}$$

Example:

$$2^{-2} = \frac{1}{2^{2}} = \frac{1}{4}$$

Negative Bases

Negative bases result in negative products.

$$-a^{x}$$

Example:

$$-64^{\frac{1}{2}} = -\sqrt[2]{64^{1}} = -8$$

Exponent Rules

Negative Bases with Parentheses

(1) If the exponent is odd, the product will be negative.
(2) If the exponent is even, the product will be positive.

$$(-a)^{x} \quad \text{OR} \quad (-a)^{-x}$$

Example (1):

$$(-7)^{3} = -7 \cdot -7 \cdot -7 = -343$$

Example (2):

$$(-7)^{2} = -7 \cdot -7 = 49$$

Fractional Bases

Fractional bases should be converted so that the numerator and denominator both contain the exponent (Power of a Fraction Rule).

$$\left(\frac{a}{b}\right)^{x} = \frac{a^{x}}{b^{x}}$$

Example:

$$\left(\frac{3}{2}\right)^{2} = \frac{3^{2}}{2^{2}} = \frac{9}{4}$$

Exponent Rules

Multiplying Exponents with the Same Base

If the bases are the same, the exponents are added together (Product Rule).

$$a^x \cdot a^y = a^{x+y}$$

Example:

$$3^2 \cdot 3^3 = 3^{2+3} = 3^5 = 243$$

Dividing Exponents with the Same Base

If the bases are the same, the exponents are subtracted (Quotient Rule).

$$a^x \div a^y = a^{x-y}$$

Example:

$$3^4 \div 3^2 = 3^{4-2} = 3^2 = 9$$

Bases with Multiple Terms within Parentheses

Each of the terms within the base can individually receive the exponent (Power of a Product Rule).

$$(ab)^x = a^x \cdot b^x$$

Example:

$$(yz)^x = y^x \cdot z^x$$

Exponent Rules

One as an Exponent

If the exponent is one, the product is the same as the base.

$$a^1 = a$$

Example:

$$3^1 = 3$$

Zero as an Exponent

If the exponent is zero, the product is one (Zero Exponent Rule).
*Note: If the base is zero, the product is undefined.

$$a^0 = 1$$

Example:

$$3^0 = 1$$

Exponents of Exponents

If an exponent contains its own exponent, the two exponents should be multiplied (Power Rule).

$$(a^x)^y = a^{x \cdot y}$$

Example:

$$(2^2)^2 = 2^{2 \cdot 2} = 16$$

Exponent Rules

Fractional Bases with Negative Exponents

Fractional bases with negative exponents can be changed into their reciprocal with positive exponents (Negative Exponent Rule and Power of a Fraction Rule).

$$\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^{x} = \frac{b^{x}}{a^{x}}$$

Example:

$$\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^{2} = \frac{2^{2}}{3^{2}} = \frac{4}{9}$$

Using a Calculator

In some cases, it isn't possible to determine the product of a number in exponential notation without a calculator.

Example:

$$8^{2.35} = 132.513\ldots$$

Example:

$$1.25^{3.1} = 1.997\ldots$$

Example:

$$200^{-0.6} = 0.041\ldots$$

Exponent Rules

$a \neq 0 \quad b \neq 0$

Product Rule

$$a^x \cdot a^y = a^{x+y}$$

Quotient Rule

$$a^x \div a^y = a^{x-y}$$

Power Rule

$$(a^x)^y = a^{x \cdot y}$$

Power of a Product Rule

$$(ab)^x = a^x \cdot b^x$$

Zero Exponent Rule

$$a^0 = 1$$

Fractional Exponent Rule

$$a^{\frac{x}{y}} = \sqrt[y]{a^x}$$

Negative Exponent Rule

$$a^{-x} = \frac{1}{a^x}$$

Power of a Fraction Rule

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$