Derivatives of Inverse Trigonometric Functions

Name: _____   Date: _____

Question 1

Find the derivative

I. $y = \cos^{-1}(x^2)$

II. $y = \sin^{-1}(1 - x)$

III. $y = \ln(\tan^{-1} x)$

IV. $y = \cot^{-1}\sqrt{x}$

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V. $y = \sec^{-1} 5x$

VI. $y = \csc^{-1}(e^x)$

Question 2

Find the derivative at the point

I. $3\tan^{-1}x + \sin^{-1}y = \dfrac{\pi}{4}$ (1 , -1)

Derivatives of Inverse Trigonometric Functions

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Question 1

Find the derivative

I. $y = \cos^{-1}(x^2)$

$$-\frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x \longrightarrow \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{d}{dx} x^2 \longrightarrow 2x$$

II. $y = \sin^{-1}(1-x)$

$$\frac{1}{\sqrt{1-(1-x)^2}} \cdot -1 \longrightarrow \frac{-1}{\sqrt{2x-x^2}}$$

$$\frac{d}{dx} 1 \longrightarrow 0$$

$$-\frac{d}{dx} x \longrightarrow -1$$

III. $y = \ln(\tan^{-1}x)$

$$\frac{1}{\tan^{-1}x} \quad \frac{1}{1+(x)^2} \cdot 1 \longrightarrow \frac{1}{(\tan^{-1}x)\cdot(1+x^2)}$$

$$\frac{d}{dx} x \longrightarrow 1$$

IV. $y = \cot^{-1}\sqrt{x} \longrightarrow y = \cot^{-1}(x)^{1/2}$

$$-\frac{1}{1+((x)^{1/2})^2} \cdot \frac{1}{2x^{1/2}} \longrightarrow \frac{-1}{(1+x)\cdot(2x^{1/2})} \longrightarrow \frac{-1}{(1+x)\cdot(2\sqrt{x})}$$

$$\frac{d}{dx} x^{1/2} \longrightarrow (1/2)x^{-1/2} \longrightarrow \frac{1}{2x^{1/2}}$$

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V. $y = \sec^{-1} 5x$

$$\frac{1}{|5x|\sqrt{(5x)^2-1}}\cdot 5 \longrightarrow \frac{5}{|5x|\sqrt{25x^2-1}} \longrightarrow \frac{1}{|x|\sqrt{25x^2-1}}$$

$$5\frac{d}{dx}x \longrightarrow 5$$

VI. $y = \csc^{-1}(e^x)$

$$-\frac{1}{|e^x|\sqrt{(e^x)^2-1}}\cdot e^x \longrightarrow -\frac{e^x}{|e^x|\sqrt{e^{2x}-1}} \longrightarrow \frac{-1}{\sqrt{e^{2x}-1}}$$

$$\frac{d}{dx}e^x \longrightarrow e^x$$

Question 2

Find the derivative at the point

I. $3\tan^{-1}x + \sin^{-1}y = \dfrac{\pi}{4}$ $(1, -1)$

$$3\cdot\frac{1}{1+(x)^2}\cdot 1 \longrightarrow \frac{3}{1+(x)^2}$$

$$\frac{d}{dx}x \longrightarrow 1$$

$$\frac{1}{\sqrt{1-(y)^2}}\cdot 1y' \longrightarrow \frac{1y'}{\sqrt{1-(y)^2}}$$

$$\frac{d}{dx}y \longrightarrow 1y'$$

$$\frac{d}{dx}\frac{\pi}{4} \longrightarrow 0$$

$$\frac{3}{1+(x)^2} + \frac{1y'}{\sqrt{1-(y)^2}} = 0 \longrightarrow \frac{1y'}{\sqrt{1-(y)^2}} = -\frac{3}{1+(x)^2} \longrightarrow \frac{-\dfrac{3}{1+(x)^2}}{\dfrac{1}{\sqrt{1-(y)^2}}}$$

$$\Big\downarrow$$

$$0 \longleftarrow -\frac{3\sqrt{1-(-1)^2}}{1+(1)^2} \longleftarrow -\frac{3\sqrt{1-y^2}}{1+x^2}$$