Derivatives of Trigonometric Functions

Name: _____   Date: _____

Question 1

Find the derivative

I. $y = -10x + 3\cos x$

II. $y = \sin x \tan x$

III. $y = \csc x - 4\sqrt{x} + \dfrac{7}{e^x}$

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IV. $y = \dfrac{\cos x}{\sin^2 x}$

V. $y = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$

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VI. $y = \dfrac{1 + \csc x}{1 - \csc x}$

VII. $y = (1 + \csc x) \cos x$

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Name: __**Key**__   Date: _____

Question 1

Find the derivative

I. $y = -10x + 3\cos x$

$$\underset{h \to 0}{\text{Lim}} \quad \frac{(-10(x+h)) - (-10(x))}{h} \longrightarrow \frac{-10\cancel{h}}{\cancel{h}} \longrightarrow -10$$

$$3\frac{dy}{dx}\cos x \longrightarrow 3 \cdot -\sin x \longrightarrow -3\sin x$$

$$-10 + -3\sin x \longrightarrow -10 - 3\sin x$$

II. $y = \sin x \tan x$

$$\frac{dy}{dx}\sin x \longrightarrow \cos x$$

$$\frac{dy}{dx}\tan x \longrightarrow \sec^2 x$$

$$\sin x \cdot \sec^2 x + \tan x \cdot \cos x \longrightarrow \sin x \sec^2 x + (\sin x / \cancel{\cos x}) \cdot \cancel{\cos x} \longrightarrow \sin x \sec^2 x + \sin x \longrightarrow \sin x(\sec^2 x + 1)$$

III. $y = \csc x - 4\sqrt{x} + \dfrac{7}{e^x} \longrightarrow y = \csc x + -4\sqrt{x} + \dfrac{7}{e^x}$

$$\frac{dy}{dx}\csc x \longrightarrow -\csc x \cot x$$

$$-4\frac{dy}{dx}x^{1/2} \longrightarrow -4 \cdot (1/2)x^{-1/2} \longrightarrow -2x^{-1/2}$$

$$7\frac{dy}{dx}e^{-x} \longrightarrow 7 \cdot -e^{-x} \longrightarrow -7e^{-x}$$

$$-\csc x \cot x + -2x^{-1/2} + -7e^{-x} \longrightarrow -\csc x \cot x - 2x^{-1/2} - 7e^{-x}$$

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IV. $\quad y = \dfrac{\cos x}{\sin^2 x}$

$\dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\sin x} \longrightarrow \cot x \cdot \csc x$

$\dfrac{dy}{dx} \cot x \longrightarrow -\csc^2 x$

$\dfrac{dy}{dx} \csc x \longrightarrow -\csc x \cot x$

$\cot x \cdot -\csc x \cot x + \csc x \cdot -\csc^2 x \longrightarrow -\cot^2 x \csc x + -\csc^3 x \longrightarrow -\csc x \cot^2 x + -\csc^3 x \longrightarrow -\csc x (\cot^2 x + \csc^2 x)$

V. $\quad y = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$

$\dfrac{4}{\cos x} \longrightarrow 4 \sec x$

$\dfrac{1}{\tan x} \longrightarrow \cot x$

$4 \dfrac{dy}{dx} \sec x \longrightarrow 4 \sec x \tan x$

$\dfrac{dy}{dx} \cot x \longrightarrow -\csc^2 x$

$4 \sec x \tan x + -\csc^2 x \longrightarrow 4 \sec x \tan x - \csc^2 x$

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VI. $\quad y = \dfrac{1 + \csc x}{1 - \csc x}$

$\dfrac{dy}{dx}\ 1 \longrightarrow 0$

$\dfrac{dy}{dx}\ \csc x \longrightarrow -\csc x \cot x$

$\dfrac{dy}{dx}\ 1 \longrightarrow 0$

$\dfrac{dy}{dx}\ -\csc x \longrightarrow \csc x \cot x$

$$\dfrac{((1 - \csc x) \cdot (-\csc x \cot x)) - ((1 + \csc x) \cdot (\csc x \cot x))}{(1 - \csc x)^2}$$

$\downarrow$

$$-\dfrac{2 \csc x \cot x}{(1 - \csc x)^2}$$

VII. $\quad y = (1 + \csc x) \cos x \longrightarrow y = \cos x + \csc x \cos x \longrightarrow y = \cos x + \cot x$

$\dfrac{dy}{dx}\ \cos x \longrightarrow -\sin x$

$\dfrac{dy}{dx}\ \cot x \longrightarrow -\csc^2 x$

$-\sin x + -\csc^2 x \longrightarrow -\sin x - \csc^2 x$