

Linearization and Differentials

Name: _____

Date: _____

Question 1

Find the linearization

I. $f(x) = x^3 - 2x + 3$ $a = 2$

II. $f(x) = \tan x$ $a = \pi$

Question 2

Find the linearization (approximation)

I. $f(x) = x^2 + 2x$ $a = 0.1$

II. $f(x) = 1 + x$

$a = 8.1$

Question 3

Find dy

I. $y = \cos(x^2)$

II. $y = x^3 - 3\sqrt{x}$

Question 4

$$f(x) = x^2 + 2x \quad x_0 = 1 \quad dx = 0.1$$

(A) find the change $\Delta f = f(x_0 + dx) - f(x_0)$

(B) find the value of the estimate of $df = f'(x_0) dx$

Question 5

write a differential formula for the change

I. $V = x^3$ x_0 to $x_0 + dx$

II. $S = 6x^2$ x_0 to $x_0 + dx$

$$\text{II. } f(x) = 1 + x \quad a = 8.1 \longrightarrow a = 8$$

near integer

$$f'(x) \longrightarrow 1$$

$$\frac{dy}{dx} 1 \longrightarrow 0 \quad \frac{dy}{dx} x \longrightarrow 1$$

$$L(x) = f(8) + f'(8)(x - 8) \longrightarrow 9 + 1(x - 8) \longrightarrow x + 1$$

$$f(8) = 1 + (8) = 9$$

$$f'(8) = 1$$

Question 3

Find dy

$$\text{I. } y = \cos(x^2)$$

$$\frac{dy}{dx} \cos \longrightarrow -\sin \longrightarrow -\sin(x^2)$$

$$\frac{dy}{dx} x^2 \longrightarrow 2x$$

$$-\sin(x^2) \cdot 2x \longrightarrow -2x \sin(x^2) \longrightarrow -2x \sin(x^2) dx$$

$$\text{II. } y = x^3 - 3\sqrt{x} \longrightarrow x^3 - 3(x)^{1/2}$$

$$\frac{dy}{dx} x^3 \longrightarrow 3x^2$$

$$-3 \frac{dy}{dx} (x)^{1/2} \longrightarrow -3 \cdot 1/2 (x)^{-1/2} \longrightarrow \frac{-3}{2(x)^{1/2}}$$

$$3x^2 + \frac{-3}{2(x)^{1/2}} \longrightarrow \left(3x^2 + \frac{-3}{2(x)^{1/2}} \right) dx$$

Question 4

$$f(x) = x^2 + 2x \quad x_0 = 1 \quad dx = 0.1$$

$$f'(x) = 2x + 2$$

$$\frac{dy}{dx} x^2 \longrightarrow 2x \quad 2 \frac{dy}{dx} x \longrightarrow 2$$

(A) find the change $\Delta f = f(x_0 + dx) - f(x_0)$

$$(x_0 + dx) \longrightarrow (1 + 0.1) \longrightarrow 1.1$$

$$f(1.1) \longrightarrow (1.1)^2 + 2(1.1) \longrightarrow 3.41$$

$$f(1) \longrightarrow (1)^2 + 2(1) \longrightarrow 3$$

$$3.41 - 3 = 0.41$$

(B) find the value of the estimate of $df = f'(x_0) dx$

$$f'(1) \longrightarrow 2(1) + 2 \longrightarrow 4$$

$$4 \cdot 0.1 = 0.4$$

(C) find the approximation error $|\Delta f - df|$

$$|0.41 - 0.4| = 0.01$$

Question 5

write a differential formula for the change

I. $V = x^3$ x_0 to $x_0 + dx$

$$dV = 3x_0^2 dx$$

$$\frac{dy}{dx} x^3 \longrightarrow 3x^2$$

II. $S = 6x^2$ x_0 to $x_0 + dx$

$$dS = 12x_0 dx$$

$$6 \frac{dy}{dx} x^2 \longrightarrow 12x$$