sin x / x Limits Practice

Name: _____ Date: _____

Question 1

Solve

I. $\lim_{x \to 0^-} \frac{x}{\sin 3x}$

II. $\lim_{x \to 0} \frac{\sin \sqrt{2} \cdot x}{\sqrt{2} \cdot x}$

III. $\lim_{x \to 0} \frac{\tan 2x}{x}$

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IV. $\lim_{x \to 0} \frac{2x}{\tan x}$

V. $\lim_{x \to 0} \frac{x \csc 2x}{\cos 5x}$

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VI. $\lim_{x \to 0} 6x^2(\cot x)(\csc 2x)$

VII. $\lim_{x \to 0} \frac{x + x\cos x}{\sin x \cos x}$

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VIII. $\lim_{x \to 0} \dfrac{x^2 - x + \sin x}{2x}$

IX. $\lim_{x \to 0} \dfrac{1 - \cos x}{\sin 2x}$

X. $\lim_{x \to 0} \dfrac{x - x\cos x}{\sin^2 3x}$

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XI. $\lim_{x \to 0} \dfrac{\sin(1-\cos x)}{1-\cos x}$

XII. $\lim_{x \to 0} \dfrac{\sin(\sin x)}{\sin x}$

XIII. $\lim_{x \to 0} \dfrac{\sin x}{\sin 2x}$

XIV. $\lim_{x \to 0} \dfrac{\sin 5x}{\sin 4x}$

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XV. $\lim_{x \to 0} x \cos x$

XVI. $\lim_{x \to 0} \sin x \cot 2x$

XVII. $\lim_{x \to 0} \frac{\tan 3x}{\sin 8x}$

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XVIII. $\lim_{x \to 0} \dfrac{\sin 3x \cot 5x}{x \cot 4x}$

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XIX. $\lim_{x \to 0} \dfrac{\tan x}{x^2 \cot 3x}$

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Name: **Key** Date: _____

Question 1

Solve

I. $\lim_{x \to 0^-} \frac{x}{\sin 3x} \to \lim_{x \to 0^-} \left(\frac{1}{3} \cdot \frac{3x}{\sin 3x} \right) \to \frac{1}{3} \lim_{x \to 0^-} \frac{1}{\left(\frac{\sin 3x}{3x} \right)}$

$\downarrow \ (x = 3x)$

$\frac{1}{3} \cdot 1 \to \frac{1}{3}$

II. $\lim_{x \to 0} \frac{\sin \sqrt{2} \cdot x}{\sqrt{2} \cdot x} \to 1$

$(x = \sqrt{2} \cdot x)$

III. $\lim_{x \to 0} \frac{\tan 2x}{x} \to \lim_{x \to 0} \frac{\left(\frac{\sin 2x}{\cos 2x} \right)}{x} \to \lim_{x \to 0} \frac{\sin 2x}{x \cos 2x}$

$\downarrow$

$\left(\lim_{x \to 0} \frac{1}{\cos 2x} \right) \left(\lim_{x \to 0} \frac{2 \sin 2x}{2x} \right)$

$\downarrow$

$\left(\lim_{x \to 0} \sec 2x \right) \left(\lim_{x \to 0} \frac{2 \sin 2x}{2x} \right)$

$\downarrow$

$1 \cdot 2 \to 2$

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IV. $\lim_{x \to 0} \frac{2x}{\tan x} \to 2 \lim_{x \to 0} \frac{x}{\left(\frac{\sin x}{\cos x}\right)} \to 2 \lim_{x \to 0} \frac{x \cos x}{\sin x}$

$\downarrow$

$2 \leftarrow 2 \cdot 1 \cdot 1 \leftarrow 2 \left(\lim_{x \to 0} \cos x\right)\left(\lim_{x \to 0} \frac{1}{\frac{\sin x}{x}}\right)$

V. $\lim_{x \to 0} \frac{x \csc 2x}{\cos 5x} \to \lim_{x \to 0} \left(\frac{x}{\sin 2x} \cdot \frac{1}{\cos 5x}\right)$

$\downarrow$

$\left(\frac{1}{2} \lim_{x \to 0} \frac{2x}{\sin 2x}\right)\left(\lim_{x \to 0} \frac{1}{\cos 5x}\right)$

$\downarrow$

$\left(\frac{1}{2} \cdot 1\right)\left(\lim_{x \to 0} \sec 5x\right)$

$\downarrow$

$\left(\frac{1}{2} \cdot 1\right)(1) \to \frac{1}{2}$

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VI. $\lim_{x \to 0} 6x^2(\cot x)(\csc 2x) \to \lim_{x \to 0} \frac{6x^2 \cos x}{\sin x \sin 2x}$

$$\downarrow$$

$$\lim_{x \to 0} \left(3\cos x \cdot \frac{x}{\sin x} \cdot \frac{2x}{\sin 2x} \right)$$

$$\downarrow$$

$$3 \cdot 1 \cdot 1 \to 3$$

VII. $\lim_{x \to 0} \frac{x + x\cos x}{\sin x \cos x} \to \lim_{x \to 0} \left(\frac{x}{\sin x \cos x} + \frac{x \cos x}{\sin x \cos x} \right)$

$$\downarrow$$

$$\lim_{x \to 0} \left(\frac{x}{\sin x} \cdot \frac{1}{\cos x} \right) + \lim_{x \to 0} \frac{x}{\sin x}$$

$$\downarrow$$

$$\lim_{x \to 0} \left(\frac{1}{\frac{\sin x}{x}} \right) \cdot \lim_{x \to 0} \left(\frac{1}{\cos x} \right) + \lim_{x \to 0} \left(\frac{1}{\frac{\sin x}{x}} \right)$$

$$\downarrow$$

$$1 \cdot 1 + 1 \to 2$$

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VIII. $\lim_{x \to 0} \frac{x^2 - x + \sin x}{2x} \to \lim_{x \to 0} \left(\frac{x}{2} - \frac{1}{2} + \frac{1}{2}\left(\frac{\sin x}{x}\right)\right) \to 0 - \frac{1}{2} + \frac{1}{2}(1)$

$\downarrow$

0

IX. $\lim_{x \to 0} \frac{1 - \cos x}{\sin 2x} \to \lim_{x \to 0} \frac{(1 - \cos x)(1 + \cos x)}{(2 \sin x \cos x)(1 + \cos x)} \to \lim_{x \to 0} \frac{1 - \cos^2 x}{(2 \sin x \cos x)(1 + \cos x)}$

$\downarrow$

$0 \leftarrow \lim_{x \to 0} \frac{\sin x}{(2 \cos x)(1 + \cos x)} \leftarrow \lim_{x \to 0} \frac{\sin^2 x}{(2 \sin x \cos x)(1 + \cos x)}$

X. $\lim_{x \to 0} \frac{x - x \cos x}{\sin^2 3x} \to \lim_{x \to 0} \frac{x(1 - \cos x)}{\sin^2 3x} \to \lim_{x \to 0} \frac{\frac{x(1 - \cos x)}{9x^2}}{\frac{\sin^2 3x}{9x^2}}$

$\downarrow$

$\lim_{x \to 0} \frac{\frac{1 - \cos x}{9x}}{\left(\frac{\sin 3x}{3x}\right)^2} \leftarrow \frac{\frac{1}{9} \lim_{x \to 0} \left(\frac{1 - \cos x}{x}\right)}{\lim_{x \to 0} \left(\frac{\sin 3x}{3x}\right)^2}$

$\downarrow$

$\frac{\frac{1}{9}(0)}{1^2} \to 0$

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XI. $\lim_{x \to 0} \dfrac{\sin(1-\cos x)}{1-\cos x} \to 1$

$(x = 1-\cos x)$

XII. $\lim_{x \to 0} \dfrac{\sin(\sin x)}{\sin x} \to 1$

$(x = \sin x)$

XIII. $\lim_{x \to 0} \dfrac{\sin x}{\sin 2x} \to \lim_{x \to 0} \left(\dfrac{\sin x}{\sin 2x} \cdot \dfrac{2x}{2x} \right)$

$\downarrow$

$\dfrac{1}{2} \lim_{x \to 0} \left(\dfrac{\sin x}{x} \cdot \dfrac{2x}{\sin 2x} \right)$

$\downarrow$

$\dfrac{1}{2} \cdot 1 \cdot 1 \to \dfrac{1}{2}$

XIV. $\lim_{x \to 0} \dfrac{\sin 5x}{\sin 4x} \to \lim_{x \to 0} \left(\dfrac{\sin 5x}{\sin 4x} \cdot \dfrac{4x}{5x} \cdot \dfrac{5}{4} \right)$

$\downarrow$

$\dfrac{5}{4} \lim_{x \to 0} \left(\dfrac{\sin 5x}{5x} \cdot \dfrac{4x}{\sin 4x} \right)$

$\downarrow$

$\dfrac{5}{4} \cdot 1 \cdot 1 \to \dfrac{5}{4}$

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XV. $\lim_{x \to 0} \; x \cos x \;\to\; 0 \cdot \cos(0) \;\to\; 0$

XVI. $\lim_{x \to 0} \sin x \cot 2x \to \lim_{x \to 0} \sin x \dfrac{\cos 2x}{\sin 2x} \to \lim_{x \to 0} \sin x \dfrac{\cos 2x}{2 \sin x \cos x}$

$\downarrow$

$\lim_{x \to 0} \dfrac{\cos 2x}{2 \cos x} \leftarrow \dfrac{1}{2}$

XVII. $\lim_{x \to 0} \dfrac{\tan 3x}{\sin 8x} \to \lim_{x \to 0} \left(\dfrac{\sin 3x}{\cos 3x} \cdot \dfrac{1}{\sin 8x} \right)$

$\downarrow$

$\lim_{x \to 0} \left(\dfrac{\sin 3x}{\cos 3x} \cdot \dfrac{1}{\sin 8x} \cdot \dfrac{8x}{3x} \cdot \dfrac{3}{8} \right)$

$\downarrow$

$\dfrac{3}{8} \lim_{x \to 0} \left(\dfrac{1}{\cos 3x} \right) \left(\dfrac{\sin 3x}{3x} \right) \left(\dfrac{8x}{\sin 8x} \right)$

$\downarrow$

$\dfrac{3}{8} \lim_{x \to 0} \left(\sec 3x \right) \left(\dfrac{\sin 3x}{3x} \right) \left(\dfrac{8x}{\sin 8x} \right)$

$\downarrow$

$\dfrac{3}{8} \cdot 1 \cdot 1 \cdot 1 \;\to\; \dfrac{3}{8}$

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XVIII. $\lim_{x \to 0} \dfrac{\sin 3x \cot 5x}{x \cot 4x} \to \lim_{x \to 0} \dfrac{\sin 3x \sin 4x \cos 5x}{x \cos 4x \sin 5x}$

$\downarrow$

$\lim_{x \to 0} \left(\dfrac{\sin 3x}{x}\right)\left(\dfrac{\sin 4x}{\cos 4x}\right)\left(\dfrac{\cos 5x}{\sin 5x}\right)\left(\dfrac{3 \cdot 4 \cdot 5x}{3 \cdot 4 \cdot 5x}\right)$

$\downarrow$

$\lim_{x \to 0} \left(\dfrac{\sin 3x}{3x}\right)\left(\dfrac{\sin 4x}{4x}\right)\left(\dfrac{5x}{\sin 5x}\right)\left(\dfrac{\cos 5x}{\cos 4x}\right)\left(\dfrac{3 \cdot 4}{5}\right)$

$\downarrow$

$\lim_{x \to 0} \left(\dfrac{\sin 3x}{3x}\right)\left(\dfrac{\sin 4x}{4x}\right)\left(\dfrac{5x}{\sin 5x}\right)(\cos 5x \sec 4x)\left(\dfrac{12}{5}\right)$

$\downarrow$

$1 \cdot 1 \cdot 1 \cdot 1 \cdot \dfrac{12}{5} \to \dfrac{12}{5}$

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XIX. $\lim_{x \to 0} \dfrac{\tan x}{x^2 \cot 3x} \to \lim_{x \to 0} \dfrac{\frac{\sin x}{\cos x}}{x^2 \frac{\cos 3x}{\sin 3x}} \to \lim_{x \to 0} \dfrac{\sin x \sin 3x}{x^2 \cos x \cos 3x}$

$\downarrow$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin 3x}{3x}\right)\left(\frac{3}{\cos x \cos 3x}\right)$$

$\downarrow$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin 3x}{3x}\right)(3 \sec x \sec 3x)$$

$\downarrow$

$1 \cdot 1 \cdot (3 \cdot 1 \cdot 1) \to 3$

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