Implicit Differentiation

Name: _____ Date: _____

Question 1

Find the derivative

I. $x^2y + xy^2 = 6$

II. $x^3 + y^3 = 18xy$

III. $2xy + y^2 = x + y$

Implicit Differentiation

IV. $x^3 - xy + y^3 = 1$

V. $x^2(x - y)^2 = x^2 - y^2$

Implicit Differentiation

VI. $(3xy + 7)^2 = 6y$

VII. $y^2 = \frac{x - 1}{x + 1}$

Implicit Differentiation

VIII. $xy = \cot(xy)$

IX. $e^{2x} = \sin(x + 3y)$

Implicit Differentiation

X. $x^2 + y^2 = 1$

Question 2

Find the line that is normal to the curve

I. $x^2y^2 = 9 \quad (-1, 3)$

Implicit Differentiation

Name: **y' tracker examples** Date: _____

Question 1

Find the derivative

I. $x^2y + xy^2 = 6 \rightarrow x^2y + xy^2 = 0 \rightarrow (2xy + x^2y') + (y^2 + 2xyy') = 0 \rightarrow 2xyy' + x^2y' = -y^2 - 2xy \rightarrow \frac{-y^2 - 2xy}{2xy + x^2}$

separate y' terms

remove y' tracker

$\frac{d}{dx} x^2 \rightarrow 2x \quad \frac{d}{dx} y \rightarrow 1y' \quad 2x \cdot y + 1y' \cdot x^2 \rightarrow 2xy + x^2y'$

$\frac{d}{dx} x \rightarrow 1 \quad \frac{d}{dx} y^2 \rightarrow 2yy' \quad 1 \cdot y^2 + 2yy' \cdot x \rightarrow y^2 + 2xyy'$

$\frac{d}{dx} 6 \rightarrow 0$

II. $x^3 + y^3 = 18xy \rightarrow 3x^2 + 3y^2y' = 18xy' + 18y \rightarrow 3y^2y' - 18xy' = 18y - 3x^2 \rightarrow \frac{18y - 3x^2}{3y^2 - 18x} \rightarrow \frac{3(6y - x^2)}{3(y^2 - 6x)} \rightarrow \frac{6y - x^2}{y^2 - 6x}$

separate y' terms

remove y' tracker

$\frac{d}{dx} x^3 \rightarrow 3x^2$

$\frac{d}{dx} y^3 \rightarrow 3y^2y'$

$18\frac{d}{dx} x \rightarrow 18 \quad \frac{d}{dx} y \rightarrow 1y' \quad 18x \cdot 1y' + 18 \cdot y \rightarrow 18xy' + 18y$

III. $2xy + y^2 = x + y \rightarrow (2xy' + 2y) + 2yy' = 1 + 1y' \rightarrow (2xy' + 2yy') - 1y' = 1 - 2y \rightarrow \frac{1 - 2y}{2x + 2y - 1}$

separate y' terms

remove y' tracker

$2\frac{d}{dx} x \rightarrow 2 \quad \frac{d}{dx} y \rightarrow 1y' \quad 2x \cdot 1y' + 2 \cdot y \rightarrow 2xy' + 2y$

$\frac{d}{dx} y^2 \rightarrow 2yy'$

$\frac{d}{dx} x \rightarrow 1 \quad \frac{d}{dx} y \rightarrow 1y'$

Implicit Differentiation

Name: **Key** Date: _____

Question 1

Find the derivative

I. $x^2y + xy^2 = 6 \rightarrow x^2y + xy^2 = 0 \rightarrow (2xy + x^2) + (y^2 + 2xy) = 0 \rightarrow 2xy + x^2 = -y^2 - 2xy \rightarrow \frac{-y^2 - 2xy}{2xy + x^2}$

$\frac{d}{dx} x^2 \rightarrow 2x \qquad \frac{d}{dx} y \rightarrow 1 \qquad 2x \cdot y + 1 \cdot x^2 \rightarrow 2xy + x^2$

$\frac{d}{dx} x \rightarrow 1 \qquad \frac{d}{dx} y^2 \rightarrow 2y \qquad 1 \cdot y^2 + 2y \cdot x \rightarrow y^2 + 2xy$

$\frac{d}{dx} 6 \rightarrow 0$

II. $x^3 + y^3 = 18xy \rightarrow 3x^2 + 3y^2 = 18x + 18y \rightarrow 3y^2 - 18x = 18y - 3x^2 \rightarrow \frac{18y - 3x^2}{3y^2 - 18x} \rightarrow \frac{3(6y - x^2)}{3(y^2 - 6x)}$

$\downarrow$

$\frac{6y - x^2}{y^2 - 6x}$

$\frac{d}{dx} x^3 \rightarrow 3x^2$

$\frac{d}{dx} y^3 \rightarrow 3y^2$

$18\frac{d}{dx} x \rightarrow 18 \qquad \frac{d}{dx} y \rightarrow 1 \qquad 18x \cdot 1 + 18 \cdot y \rightarrow 18x + 18y$

III. $2xy + y^2 = x + y \rightarrow (2x + 2y) + 2y = 1 + 1 \rightarrow (2x + 2y) - 1 = 1 - 2y \rightarrow \frac{1 - 2y}{2x + 2y - 1}$

$2\frac{d}{dx} x \rightarrow 2 \qquad \frac{d}{dx} y \rightarrow 1 \qquad 2x \cdot 1 + 2 \cdot y \rightarrow 2x + 2y$

$\frac{d}{dx} y^2 \rightarrow 2y$

$\frac{d}{dx} x \rightarrow 1 \qquad \frac{d}{dx} y \rightarrow 1$

Implicit Differentiation

IV. $x^3 - xy + y^3 = 1 \longrightarrow 3x^2 - (x + y) + 3y^2 = 0 \longrightarrow 3x^2 - x - y + 3y^2 = 0 \longrightarrow 3y^2 - x = y - 3x^2 \longrightarrow \frac{y - 3x^2}{3y^2 - x}$

$\frac{d}{dx} x^3 \longrightarrow 3x^2$

$\frac{d}{dx} x \longrightarrow 1 \qquad \frac{d}{dx} y \longrightarrow 1 \qquad 1 \cdot x + 1 \cdot y \longrightarrow x + y$

$\frac{d}{dx} y^3 \longrightarrow 3y^2$

$\frac{d}{dx} 1 \longrightarrow 0$

V. $x^2(x - y)^2 = x^2 - y^2 \longrightarrow x^2 \cdot (x^2 - 2xy + y^2) = x^2 - y^2 \longrightarrow x^4 - 2x^3y + x^2y^2 = x^2 - y^2 \longrightarrow$

$(4x^3) + (-2x^3 + -6x^2y) + (2x^2y + 2xy^2) = 2x + -2y \longrightarrow (2x^2y) + (-2x^3) + 2y = 2x - (4x^3) - (-6x^2y) - (2xy^2)$

$\downarrow$

$\frac{2x - 4x^3 - -6x^2y - 2xy^2}{2x^2y + -2x^3 + 2y} \longleftarrow \frac{2x - 4x^3 + 6x^2y - 2xy^2}{2x^2y - 2x^3 + 2y} \longleftarrow \frac{\cancel{2}(x - 2x^3 + 3x^2y - xy^2)}{\cancel{2}(x^2y - x^3 + y)}$

$\downarrow$

$\frac{x - 2x^3 + 3x^2y - xy^2}{x^2y - x^3 + y}$

$\frac{d}{dx} x^4 \longrightarrow 4x^3 \quad -2\frac{d}{dx} x^3 \longrightarrow -6x^2 \quad \frac{d}{dx} y \longrightarrow 1 \quad \frac{d}{dx} x^2 \longrightarrow 2x \quad \frac{d}{dx} y^2 \longrightarrow 2y$

$-2x^3 \cdot 1 + y \cdot -6x^2 \longrightarrow -2x^3 + -6x^2y \qquad x^2 \cdot 2y + y^2 \cdot 2x \longrightarrow 2x^2y + 2xy^2$

$\frac{d}{dx} x^2 \longrightarrow 2x \quad -\frac{d}{dx} y^2 \longrightarrow -2y$

Implicit Differentiation

VI. $(3xy+7)^2 = 6y \longrightarrow 9x^2y^2 + 42xy + 49 = 6y \longrightarrow (18x^2y + 18xy^2) + (42x + 42y) = 6$

$\downarrow$

$18xy^2 + 42y = 6 - 18x^2y - 42x \longleftarrow \dfrac{18xy^2 + 42y}{6 - 18x^2y - 42x} \longleftarrow \dfrac{\cancel{6}(3xy^2 + 7y)}{\cancel{6}(1 - 3x^2y - 7x)}$

$\downarrow$

$$\frac{3xy^2 + 7y}{1 - 3x^2y - 7x}$$

$9 \dfrac{d}{dx} x^2 \longrightarrow 18x \qquad \dfrac{d}{dx} y^2 \longrightarrow 2y$

$9x^2 \cdot 2y + y^2 \cdot 18x \longrightarrow 18x^2y + 18xy^2$

$42 \dfrac{d}{dx} x \longrightarrow 42 \qquad \dfrac{d}{dx} y \longrightarrow 1$

$42x \cdot 1 + y \cdot 42 \longrightarrow 42x + 42y$

$\dfrac{d}{dx} 49 \longrightarrow 0$

$6 \dfrac{d}{dx} y \longrightarrow 6$

VII. $y^2 = \dfrac{x-1}{x+1} \longrightarrow 2y = \dfrac{(x+1)(1) - (x-1)(1)}{(x+1)^2} \longrightarrow 2y = \dfrac{(x+1) + (-x+1)}{(x+1)^2} \longrightarrow 2y = \dfrac{2}{(x+1)^2} \longrightarrow \dfrac{\cancel{2}}{\cancel{2}y(x+1)^2}$

$\downarrow$

$$\frac{1}{y(x+1)^2}$$

$\dfrac{d}{dx} y^2 \longrightarrow 2y$

$\dfrac{d}{dx} x \longrightarrow 1 \qquad \dfrac{d}{dx} -1 \longrightarrow 0$

$\dfrac{d}{dx} x \longrightarrow 1 \qquad \dfrac{d}{dx} 1 \longrightarrow 0$

Implicit Differentiation

VIII. $xy = \cot(xy) \longrightarrow x + y = -x\csc^2(xy) + -y\csc^2(xy) \longrightarrow x - -x\csc^2(xy) = -y\csc^2(xy) - y \longrightarrow \dfrac{-y\csc^2(xy) - y}{x + x\csc^2(xy)}$

$$\downarrow$$

$$\dfrac{-y}{x} \longleftarrow \dfrac{-y(\csc^2(xy) + 1)}{x(1 + \csc^2(xy))}$$

$\dfrac{d}{dx} x \longrightarrow 1 \qquad \dfrac{d}{dx} y \longrightarrow 1 \qquad x \cdot 1 + y \cdot 1 \longrightarrow x + y$

$\dfrac{d}{dx} \cot(xy) \longrightarrow -\csc^2(xy) \quad \dfrac{d}{dx} x \longrightarrow 1 \qquad \dfrac{d}{dx} y \longrightarrow 1 \qquad x \cdot 1 + y \cdot 1 \longrightarrow x + y$

$-\csc^2(xy) \cdot (x + y) \longrightarrow -x\csc^2(xy) + -y\csc^2(xy)$

IX. $e^{2x} = \sin(x + 3y) \longrightarrow 2e^{2x} = \cos(x + 3y) + 3\cos(x + 3y) \longrightarrow 2e^{2x} - \cos(x + 3y) = 3\cos(x + 3y)$

$$\downarrow$$

$$\dfrac{2e^{2x} - \cos(x + 3y)}{3\cos(x + 3y)}$$

$\dfrac{d}{dx} e^{2x} \longrightarrow 2e^{2x}$

$\dfrac{d}{dx} \sin(x + 3y) \longrightarrow \cos(x + 3y)$

$\dfrac{d}{dx} x \longrightarrow 1 \quad 3\dfrac{d}{dx} y \longrightarrow 3$

$\cos(x + 3y) \cdot (1 + 3) \longrightarrow 1\cos(x + 3y) + 3\cos(x + 3y)$

Implicit Differentiation

X. $x^2 + y^2 = 1 \longrightarrow 2x + 2y = 0 \longrightarrow 2y = -2x \longrightarrow \frac{-2x}{2y} \longrightarrow \frac{-x}{y}$

$\frac{d}{dx} x^2 \longrightarrow 2x \qquad \frac{d}{dx} y^2 \longrightarrow 2y$

$\frac{d}{dx} 1 \longrightarrow 0$

Question 2

Find the line that is normal to the curve

I. $x^2y^2 = 9 \quad (-1, 3)$

$\frac{d}{dx} x^2 \longrightarrow 2x \qquad \frac{d}{dx} y^2 \longrightarrow 2y \qquad x^2 \cdot 2y + y^2 \cdot 2x \longrightarrow 2x^2y + 2xy^2$

$\frac{d}{dx} 9 \longrightarrow 0$

$x^2y^2 = 9 \longrightarrow 2x^2y + 2xy^2 = 0 \longrightarrow 2x^2y = -2xy^2 \longrightarrow \frac{-2xy^2}{2x^2y} \longrightarrow \frac{-y}{x}$

slope of tangent line :

$\frac{-3}{-1} \longrightarrow 3$

tangent line :

$y - (3) = (3)(x - (-1))$

normal line :

$y - (3) = (-1 / 3)(x - (-1))$